Stability of infinite slopes under transient partially saturated seepage conditions

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1 Predictions of the location and timing of rainfall-induced shallow landslides is desired by organizations responsible for hazard management and warnings. However, hydrologic and mechanical processes in the vadose zone complicate such predictions. Infiltrating rainfall must typically pass through an unsaturated layer before reaching the irregular and usually discontinuous shallow water table. This process is dynamic and a function of precipitation intensity and duration, the initial moisture conditions and hydrologic properties of the hillside materials, and the geometry, stratigraphy, and vegetation of the hillslope. As a result, pore water pressures, volumetric water content, effective stress, and thus the propensity for landsliding vary over seasonal and shorter time scales. We apply a general framework for assessing the stability of infinite slopes under transient variably saturated conditions. The framework includes profiles of pressure head and volumetric water content combined with a general effective stress for slope stability analysis. The general effective stress, or suction stress, provides a means for rigorous quantification of stress changes due to rainfall and infiltration and thus the analysis of slope stability over the range of volumetric water contents and pressure heads relevant to shallow landslide initiation. We present results using an analytical solution for transient infiltration for a range of soil texture and hydrological properties typical of landslide-prone hillslopes and show the effect of these properties on the timing and depth of slope failure. We follow by analyzing field-monitoring data acquired prior to shallow landslide failure of a hillside near Seattle, Washington, and show that the timing of the slide was predictable using measured pressure head and volumetric water content and show how the approach can be used in a forward manner using a numerical model for transient infiltration.


1. Introduction

2 Shallow landslides are important erosion processes and are potentially lethal hazards in mountainous areas. They involve the upper few meters of unconsolidated surficial materials (i.e., soil or regolith) and dominate sediment transport in some hillslope environments [e.g., Dietrich and Dunne, 1978; Caine and Swanson, 1989; Trustrum et al. 1999]. Because shallow landslides can initiate potentially destructive debris flows [e.g., Iverson et al., 1997] they present a hazard to human activities [e.g., Sidle and Ochiai, 2006].

3 Change in pore water pressure resulting from rainfall onto hillslopes is a dynamic physical process that can provoke slope failure in hillslope materials. Field monitoring of rainfall and pore water response as well as theoretical and physical models have provided insight into the timing and mechanisms of landslides induced by rainfall [e.g., Reid et al., 1997; Torres et al., 1998; Ebel et al., 2007; Montgomery et al., 2009; Borja and White, 2010]. Analysis of slope stability under positive pore water conditions in which the soil or regolith is saturated is fundamental to soil mechanics and geotechnical engineering. However, theoretical and analytical results have advanced the hypothesis that in some hillslope environments shallow landsliding is possible absent positive pore water pressures [e.g., Morgenstern and de Matos, 1975; Wolle and Hachich, 1989; Rahardjo et al., 1996; Springman et al., 2003; Collins and Znidarcic, 2004; Lu and Godt, 2008]. Recent field monitoring results seem to confirm that shallow landslides can be initiated by changes in pore water pressures in the unsaturated regime [Godt et al., 2009]. Assumption of positive pore water pressures is necessary to apply Terzaghi’s [1943] effective stress principle. This restrictive assumption hinders assessment of slope stability leading to overly conservative estimates of shallow landslide susceptibility.

4 Instability under partially saturated conditions is more likely to occur on steep slopes that have been disturbed by natural or human activities that have altered the strength of hillside materials or increased the inclination of the slopes. In such settings, the effect of disturbance is to reduce the
change in effective stress needed to cause instability. After disturbance, steep hillslopes that were previously stable even when heavy rainfall generated positive pore water pressures may now be susceptible to instability under partially saturated conditions. Examples of disturbance that alter slope stability are wildfire [e.g., Jackson and Roering, 2009], industrial forest practices [Montgomery et al., 2000], conversion of forests to pasture [Glade, 2003], and large-scale slope deformation, which may initiate smaller-scale instabilities [e.g., Imaiiumi et al., 2006]. Slopes oversteepened by active uplift, coastal erosion and bluff retreat [e.g., Godt et al., 2009] or excavated for road construction [e.g., Higgins and Modeer, 1996] may also be susceptible to slope failure under partially saturated conditions.

[5] The close link between rainfall characteristics and shallow landslide initiation is well documented [e.g., Caine, 1980; Godt et al., 2006] and quantitative understanding of this link is gained by analyzing the transient pore water response to rainfall and consequent changes in stability [e.g., Reid, 1994]. Steady state flow models [e.g., Montgomery and Dietrich, 1994; Lu and Godt, 2008] may provide insight into the pore water conditions at the time of slope failure, but little information on the rainfall needed to reach that point. Reduction of the problem to steady conditions yields a comparison of a steady infiltration flux to hydraulic conductivity, \( K \), but requires neglecting the transient storage of water in the unsaturated zone. Thus, the nonlinear dependence of the pore water response to rainfall is ignored making quantitative assessment of rainfall patterns and slope stability unreliable.

[6] In partially saturated soils, changes in volumetric water content \( \theta \) resulting from rainfall are accompanied by changes in the pressure head \( \psi \), which is described by the soil water characteristic curve \( C(\psi) = d\theta/d\psi \), is also pressure head dependent and describes the available storage in the partially saturated soil. Soil water diffusivity, \( D(\psi) = K(\psi)/C(\psi) \) is the relative amount of storage as a function of pressure-dependent hydraulic conductivity [Freeze and Cherry, 1979]. Iverson [2000] showed that for the limiting case of nearly saturated hillslopes, the dominant direction of strong pore pressure transmission resulting from rainfall can be assessed using two time scales. Assuming a reference soil water diffusivity, \( D_0 \), in an isotropic, homogeneous hillslope, the time scale for strong pore pressure diffusion normal to the slope is \( H^2/D_0 \) where \( H \) is the slope normal depth. The time scale for strong pore pressure diffusion parallel to the slope to some point located below a catchment with contributing area \( A \) is \( A\theta/D_0 \).

[7] In what follows, we describe the form of effective stress in variably saturated materials and present results for four hypothetical hillslopes using an analytical solution for transient infiltration [Srivastava and Yeh, 1991; Baum et al., 2010] to examine the variation in timing and depth of potential instability above the water table with material hydrologic properties. Here, we use the term “infiltration” to describe the flow of water across the ground surface and into the underlying soil [Freeze and Cherry, 1979, p. 211]. The four hillslopes are representative of the range of materials for which shallow landslides may occur under partially saturated conditions. We then apply the framework to a case study where pressure heads and water contents were monitored leading up to the occurrence of a landslide at the field site. These data are then used to calibrate a numerical solution to the governing equations of variably saturated flow to illustrate how the approach could be applied to forecast landslides.

2. Effective Stress in Variably Saturated Hillslopes

[8] Effective stress in partially saturated soil differs from effective stress in saturated soils [e.g., Terzaghi, 1943] in that in partially saturated soils, effective stress is not the difference between total stress and pore water pressure, but rather the difference between total stress and some function of negative pore water pressure head or soil suction. One form of this function proposed by Lu and Likos [2004, 2006] is the suction stress characteristic curve. The suction stress characteristic curve describes the saturation-dependent macroscopic stress that tends to hold particles together in partially saturated soil. This stress can vary from a few kPa in sands to several hundred kPa in silts and clays [Lu and Godt, 2008]. The generalized form of effective stress \( \sigma' \) for variably saturated soils is given by [Lu et al., 2010]:

\[
\sigma' = (\sigma - u_w) - \sigma_n
\]

(1)

where \( u_w \) is the pore air pressure, \( \sigma \) is the total stress, and suction stress, \( \sigma_n \) is given by

\[
\sigma_n = \frac{\theta - \theta_0}{\theta_0 - \theta_1} (u_w - u_0) = -S_e (u_w - u_0)
\]

(2)

where \( \theta \) is the volumetric water content, \( \theta_0 \) is the residual volumetric water content, \( \theta_1 \) is the saturated volumetric water content, \( S_e \) is the effective degree of saturation, and \( u_0 \) is pore water pressure. Lu and Likos [2004] and Lu et al. [2010] show that suction stress can be written solely as a function of the effective degree of saturation or matric suction \( (u_w - u_0) \):

\[
\sigma' = -S_e \left( \frac{u_w - u_0}{\alpha (u_w - u_0)} \right)^\frac{1}{\alpha - 1}
\]

(3)

\[
\sigma' = -\frac{(u_w - u_0)}{\left(1 + [\alpha (u_w - u_0)]^n \right)^{(n-1)/n}}
\]

(4)

where \( \alpha \) and \( n \) are parameters defined in two commonly used soil water characteristic models [van Genuchten, 1980]. Note that Terzaghi’s effective stress \( \sigma' = \sigma - u_w \) is
recovered from equation (1) for saturated materials. Below, we use these equations to extend the infinite slope stability analysis to variably saturated conditions.

3. Infinite Slope Stability

Infinite slope stability analyses \cite{Taylor,1948} are statically determinate and are therefore convenient for examining the role of infiltration and consequent transient changes in the distribution of soil moisture on stability \cite[e.g.,][]{Iverson,2000}. Slope failure is modeled as the ratio of resisting Coulomb basal friction to gravitationally driven downslope stress. This ratio, $F$, is called the factor of safety. Instability is predicted when the driving stress exceeds the resisting friction. Infinite slope stability analysis is appropriate for landslides in which the failure and ground surfaces are parallel and the slide thickness is relatively small compared to the slide length, and where the local topographic curvature is small. For transient infiltration conditions, the factor of safety $F$ at depth $Z$ below the ground surface is given by \cite{Lu_and_Godt,2008}

$$F(Z,t) = \tan \phi'(Z) + \frac{c' + \sigma'(Z,t) \tan \phi'(Z)}{\gamma_s \sin \beta \cos \beta}$$

where $\phi'(Z)$ is a depth-dependent angle of internal friction and $c'$ is the soil cohesion, $\gamma_s$ the soil unit weight, and $\beta$ the slope angle (Figure 1). We follow \cite{Lu_and_Godt} and describe the variation of the friction angle, $\phi'(Z)$, in weathered surficial materials as a function of a depth-dependent porosity $m$ \cite[e.g.,][]{Rowe,1969; Cornforth,2005}

$$\phi' = \phi'_0 + \frac{\Delta \phi'}{\Delta m} (m_0 - m)$$

where $\phi'_0$ and $m_0$ are the friction angle and porosity at the ground surface and $\Delta \phi'$ and $\Delta m$ are the range of friction angle and porosity, respectively, within the weathering zone $z_w$ (Figure 2). The decrease in porosity with depth $Z$ results mainly from mechanical compaction and consolidation of the soil under its own weight and can be described as a power reduction equation:

$$m = m_0 - \frac{\Delta m}{1 + \frac{Z}{Z_w}}$$

Substituting equation (7) into (6) yields a form for the change in friction angle with depth:

$$\phi' = \phi'_0 + \frac{\Delta \phi'}{1 + \frac{Z}{Z_w}}$$

4. Analytical Solution for Transient Flow

The one-dimensional form of Richards equation is used to describe flow of water in the vertical direction in the unsaturated zone \cite{Freeze_and_Cherry,1979}

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z} \left[ K(\psi) \frac{\partial (\psi - z)}{\partial z} \right]$$

where $z$ is the vertical coordinate, $t$ is time, $\psi$ is the pressure head, and $\theta(\psi)$ and $K(\psi)$ are the pressure head–dependent volumetric water content and hydraulic conductivity, respectively. Equation 9a can be linearized using the exponential model of \cite{Gardner,1958} to provide an analytical solution for transient infiltration above a fixed water table \cite{Srivastava_and_Yeh,1991; Baum_et_al.,2008,2010}. The hydraulic conductivity function and soil water characteristic curve are given by

$$K(\psi) = K_S \exp(\alpha \psi)$$

$$\theta(\psi) = \theta_s + (\theta_f - \theta_s) \exp(\alpha \psi)$$

![Figure 1. Schematic cross section and definitions for a variably saturated infinite slope.](image)

![Figure 2. Relative change in dimensionless friction angle as a function of depth for a weathered zone of varying thickness, $z_w$.](image)
respectively, where $K_s$ is the saturated hydraulic conductivity and $\alpha$ is the inverse of the vertical height of the capillary fringe.

[11] Substitution of (9b) and (9c) into the one-dimensional form of Richards equation (9a) yields a linear partial differential equation in terms of $K(z,t)$

$$\frac{\alpha(\theta_i - \theta_0)}{K_s} \frac{\partial K}{\partial t} = \frac{\partial^2 K}{\partial z^2} - \alpha \frac{\partial K}{\partial z}$$  (9d)

We use solutions for a constant surface flux and a stationary water table developed by Srivastava and Yeh [1991] with the method that is recapitulated below from that described by Baum et al. [2008, 2010].

[12] The flux, $q$, at the base of the unsaturated zone, obtained by solving the linearized Richards equation, is

$$q(d_w,t) = \left\{ \begin{array}{ll}
I_z - 4(I_{zt} - I_{zt}) \exp\left[\frac{\alpha_1 d_u}{2} \exp\left(-D_e \frac{t}{4}\right)\right] \\
\sum_{m=1}^{\infty} \frac{\Lambda_m \sin(\Lambda_m \alpha_1 d_u)}{1 + \frac{\alpha_1 d_u}{2} + 2\Lambda_m^2 \alpha_1 d_u} \exp[-\Lambda_m^2 D_e \frac{t}{4}] \end{array} \right. $$  (10a)

where

$$D_e = \frac{\alpha_1 K_s}{(\theta_i - \theta_0)}$$  (10b)

is the decay constant, and $D_e/\alpha^2$ is the soil water diffusivity [Freeze and Cherry, 1979]. $D_e$ is equivalent to the nondimensional time used by Srivastava and Yeh [1991]. $d_u$ is the vertical depth to the top of the capillary fringe, $\alpha_1 d_u$ is equivalent to the nondimensional depth used by Srivastava and Yeh [1991], and the values of $\Lambda_m$ are the roots of the characteristic equation (12b), $I_z$ is the surface flux of a given intensity, and $I_{zt}$ is the steady (initial) surface flux.

[13] The hydraulic conductivity in the unsaturated zone varies with depth according to the following formula. The initial condition, $K(Z,0)$, is

$$K(Z, 0) = I_{zt} - [I_{zt} - K_0 \exp(\alpha_1 \psi_0)] \exp[-\alpha_1(d_u - Z)]$$  (11)

which reduces to $K_0 \exp(\alpha_1 \psi_0)$ at depth $Z = d_u$ (or $K_s$ at the water table if $d$ is substituted for $d_u$ so that $\psi_0 = 0$) and approaches $I_{zt}$ at the ground surface as $d_u$ becomes large. The initial condition given in equation (11) is obtained by solving the steady part of equation (9d) with specified flux, $I_{zt}$, at the ground surface and specified pressure head, $\psi_0$, at the water table. The hydraulic conductivity solution for the unsaturated zone is given as

$$K(Z, t) = I_{zt} - [I_{zt} - K_0 \exp(\alpha_1 \psi_0)] \exp[-\alpha_1(d_u - Z)]$$

$$- 4(I_{zt} - I_{zt}) \exp\left[\frac{\alpha_1 Z}{2} \exp\left(-D_e \frac{t}{4}\right)\right]$$

$$\sum_{m=1}^{\infty} \frac{\sin(\Lambda_m \alpha_1 (d_u - Z)) \sin(\Lambda_m \alpha_1 d_u)}{1 + \frac{\alpha_1 d_u}{2} + 2\Lambda_m^2 \alpha_1 d_u} \exp[-\Lambda_m^2 D_e \frac{t}{4}]$$  (12a)

The values of $\Lambda_m$ are the positive roots of the pseudoperiodic characteristic equation,

$$\tan(\Lambda_m d_u) + 2\Lambda_m = 0$$  (12b)

For the values of $\Lambda_m$, a combination of bracketing, bisection, and Newton-Raphson iteration is used [Baum et al., 2008, 2010].

5. Timing of Slope Failure Above the Water Table in Steep Hillslopes

[14] We examine the timing of potential instability of a steep (45°) hillside composed of four hypothetical materials using the analytical solution for transient infiltration described in section 4. The analytical solution allows us to isolate the effects of the material hydrological properties on the timing and depth of potential instability. The materials (coarse, medium, and fine sand, and silt) have soil water and suction stress characteristics (Figure 3) and material strength properties that cover a range typical for soils that mantle steep hillsides and that are prone to shallow landsliding under partially saturated conditions (Table 1). In the hypothetical examples described below the hydrological properties of the soil are constant and the internal friction angle varies with depth according to equation (8).

[15] The soil water characteristics were calculated using the Gardner [1958] equations (9b) and (9c) and suction stress using equation (2). Peak suction stresses vary among the soils from about $-1.0$ kPa at a pressure head of about $-0.3$ m for the coarse sand, $-3.5$ kPa at a pressure head of about $-1.0$ m for the medium sand, $-5.1$ kPa at a pressure head of about $-1.4$ m for the fine sand, and $-7.2$ kPa at a pressure head of about $-2.0$ m for pressure head for the silt (Figure 3c). The pressure head response to an infiltration rate, $I_z$ equivalent to the saturated hydraulic conductivity $K_s$ was computed for a 2.0 m thick unsaturated zone above a stationary boundary with a constant pressure head $\psi = 0$ in the sand examples and a 5.0 m thick unsaturated zone in the silt example. Initial conditions were prescribed as hydrostatic for all examples, which leads to the characteristic distribution of the effective degree of saturation above the water table. These four limiting cases demonstrate the progression of instability above the water table. For hillslopes less susceptible to landsliding, such as those inclined at shallower angles, composed of stronger materials, or subject to lower infiltration rates [Lu and Godt, 2008], slope failure would likely occur in the presence of positive pore water conditions.

5.1. Coarse Sand Hillslope

[16] This hypothetical example can be considered an analog for hillslope environments where surficial materials are composed of a coarse sandy regolith with poorly developed soils lacking organic content. Such a setting is common where alteration of biotite and other minerals in granitic bedrock forms grus [Wahrhaftig, 1965], deposits of which may be tens of meters thick at ridge crests [Birkeland, 1999]. In such settings, particularly where vegetation is sparse or removed by wildfire, discrete shallow landslides may occur under both very dry and very wet conditions [e.g., Andersen et al., 1959; Durgin, 1977; Gabet, 2003].
Under initially hydrostatic conditions with a water table located 2.0 m below the ground surface (Figure 4a) the effective saturation (Figure 4b) is reduced to zero at about 1.5 m above the water table. Under hydrostatic conditions suction stress has a peak of \(-1.0 \text{ kPa}\) at 0.3 m above the water table, is zero at the water table, and diminishes to \(-0.2 \text{ kPa}\) at 1.2 m above the water table (Figure 4c). The changes in suction stress with height above the water table under hydrostatic conditions lead to factors of safety of less than unity for these initial conditions (Figure 4d) given the slope geometry, material strength and hydrologic properties selected for this example. Although unstable initial conditions are not realistic, for this case, very dry conditions lead to relatively small values of suction stress and instability compared to conditions when the soil is modestly wet. This example provides a mechanism to explain field observations of shallow landslide occurrence under very dry conditions following wildfire from the San Gabriel range front in southern California [Anderson et al., 1959; Krammes, 1963].

After 6 h of rainfall pressure heads increase to near zero at the ground surface (Figure 4a) with a consequent increase in effective saturation (Figure 4b). The effect of infiltration on the profile of suction stress is complex with large relative decrease in suction stress to a minimum of about \(-1.0 \text{ kPa}\) over the upper 1.0 m from the hydrostatic initial condition after 6 h of rainfall (Figure 4c). This decrease in suction stress leads to an increase in the factor of safety and a shift to relatively stable conditions compared to the initial state (Figure 4d). The profile is stable throughout at 12 h, but a zone of factors of safety approaching unity develops at about 0.2 m below the ground surface at this time (Figure 4d). At later times (24 and 48 h in Figures 4a and 4b), pressure heads are nearly zero throughout the profile and effective saturation is almost 1.0. Suction stress is diminished nearly completely throughout the profile and the upper 0.8 m is potentially unstable (Figure 4d).

5.2. Medium Sand Hillslope

Here, we consider the transient effects of infiltration on the stability of a hypothetical hillslope with a surficial cover of medium sand. A comparable real-world setting might be the Puget Sound region where glacial deposits mechanically weather to form a sandy colluvium a few meters thick on steep coastal hillsides [Galster and Laprade, 1991; Schulz, 2007; Troost and Booth, 2008].

Under initially hydrostatic conditions with a water table located 2.0 m below the ground surface (Figure 5a) the effective saturation decreases from 1.0 at the water table to about 0.1 at the ground surface (Figure 5b). The profile of suction stress for the medium sand (Figure 5c) has a similar form to that for the coarse-sand example (Figure 4c). Suction stress is zero at the water table, peaks at about \(-3.6 \text{ kPa}\) 1.0 m above the water table and increases to about \(-2.8 \text{ kPa}\) at the ground surface (Figure 5c). In contrast to the coarse sand, the medium sand is stable under the same initially hydrostatic conditions (Figure 5d). Factors of safety increase monotonically above the water table for the medium sand because of the contribution of suction stress. Comparison of these profiles for the coarse and medium sand cases highlights the influence of the hydrologic properties on slope stability [Lu and Godt, 2008].

After 36 h of infiltration pressure heads are nearly zero and effective saturation is close to 1.0 throughout the profile (Figures 5a and 5b). At this time suction stress peaks at \(-0.2 \text{ kPa}\) at 1.0 m above the water table (Figure 5c) and
the profile of the factor of safety indicates potential instability in a zone that extends from 0.1 to 0.5 m below the ground surface (Figure 5d). Pressure head, effective saturation, and suction stress continue to increase with continued infiltration and at 48 h the upper half of the profile is potentially unstable.

5.3. Fine Sand Hillslope

[22] In this hypothetical example we examine the effect of transient infiltration on the potential for landslide occurrence in a fine sand hillslope such as the well-drained colluvium from Pleistocene age sandstone of the Boso Peninsula in Japan where shallow landslides have resulted from heavy rainfall [Matsushi et al., 2006]. Under initially hydrostatic conditions with a water table located 2.0 m below the ground surface (Figure 6a) effective saturation decreases from 1.0 at the water table to about 0.35 at the ground surface (Figure 6b). Suction stress decreases from zero at the water table to a broad peak of about −5.2 kPa in a zone between 1.3 and 1.6 m above the water table (Figure 6c). Under initially hydrostatic conditions, factors of safety increase monotonically above the water table (Figure 6d) similar to the medium sand example described in section 5.2.

[23] After 36 h of infiltration pressure heads increase to more than −0.2 m in the upper 1.5 m of the profile (Figure 6a) and saturations are greater than 0.9 (Figure 6b). Suction stress is less than −1.0 kPa in this same zone (Figure 6c) and stability holds (Figure 6d). At 36 h pressure heads are nearly zero and the profile is almost saturated (Figures 6a and 6b), but suction stress of about −0.5 kPa persists (Figure 6c) and although factors of safety approach unity, the slope is still stable. Not until 48 h are pressure heads and suction stress of about 0.5 kPa (Figure 6c) leading to factors of safety less than unity and potential instability of the upper 0.5 m of the hillslope (Figure 6d).

5.4. Silt Hillslope

[24] Finally, we examine the timing of potential instability in a steep hillslope composed of hypothetical silt. Landslides in loess mantled landscapes, such as those that occur during heavy monsoonal rain on the Loess Plateau in northern China present a significant hazard to human activities [Derbyshire, 2001]. For this example we assume the unsaturated zone is 5 m thick above a stationary boundary with a constant pressure head \( \psi = 0 \). Initial conditions are hydrostatic and in equilibrium with the lower boundary condition (Figure 7a). The upper boundary is a prescribed flux \( I_B \) at the ground surface equivalent to the saturated hydraulic conductivity of the silt (Table 1).

[25] Under the hydrostatic initial conditions (Figure 7a) the effective saturation decreases from 1.0 at the water table to 0.08 at the ground surface (Figure 7b). Suction stress decreases with distance above the water table to about −7.2 kPa at about 2.0 m above the water table and increases to −4.0 kPa at the ground surface (Figure 7c) and the soil profile is initially stable (Figure 7d). After 20 days of infiltration pressure heads are more than −0.2 m and effective saturation is greater than about 0.9 throughout the profile (Figures 7a and 7b). Suction stress increases to −1.1 kPa at the ground surface with a minimum of about −2.0 kPa 2.5 m above the water table (Figure 7c). This change in suction stress in response to infiltration leads to potential instability in a zone near the ground surface extending from 3.6 to 4.4 m above the water table (Figure 7d).

5.5. Summary of Hypothetical Examples

[26] Table 2 provides a summary of the simulated time of failure and the failure depth for the hypothetical examples. In general, failure depth increases for the finer grained soils. This results in part from the influence of the \( \alpha \) parameter (i.e., inverse of the air entry head) in the Gardner [1958] formulation of the soil water characteristic curve (equations 9b and 9c) on both the hydrologic and mechanical properties of the materials. The maximum contribution to stability from suction stress is relatively small for the coarse sand (Figure 4c), which has a large \( \alpha \) compared to that for the fine sand (Figure 4c). Because the \( \alpha \) parameter also controls the hydraulic conductivity function (Figure 3b) it influences the simulated degree of saturation front during infiltration. The simulated degree of saturation from the coarse-sand example (Figure 4b) shows a more abrupt change with depth compared to that for the fine-sand example (Figure 6b), which in turn leads to a greater reduction in suction stress and stability near the ground surface. The presence of soil cohesion also leads to an increase in the failure depth. The silt sand example (not shown) has the same hydrological properties and response to rainfall as the medium sand case but the friction angle and cohesion used in the case study described in section 6 (Table 1). The addition of 1.1 kPa of soil cohesion results in an increase in failure depth of about 1 m over that simulated for the medium sand example without any cohesion.

[27] Because we applied a rainfall flux at the ground surface equivalent to the saturated conductivity \( K_s \) of the materials, the variation in the timing of failure for the hypothetical examples is largely a function of the variation in hydraulic conductivity among the different materials. For example, the saturated hydraulic conductivity of the coarse sand is about five times that of the fine sand, and the time to failure is longer by about the same amount. Variation in
the time to failure also results from differences in the hydraulic conductivity function at pressure heads less than zero (Figure 3b) and the failure and overall profile depths.

6. Case Study Application of the Suction Stress Concept

[28] We apply the suction stress concept to assess the potential for instability using numerical modeling results calibrated with hydrologic data collected at an instrumented hillslope along the Puget Sound near Edmonds, WA about 15 km north of Seattle (Figure 8a) where a shallow landslide occurred in the apparent absence of positive pore water pressures under partially saturated soil conditions [Godt et al., 2009]. Steep hillslopes in this area are subject to frequent shallow landslides during the winter wet season [Galster and Laprade, 1991; Schulz, 2007] when extended rainy periods lasting several days may induce shallow slides [Godt et al., 2006; Chleborad et al., 2008]. Pleistocene age glaciation, shoreline wave attack, and mass movement processes have formed steep (>30°) 50 to 100 m high coastal bluffs above many areas along the Puget Sound shoreline [Shipman, 2004]. In this area, shallow landslides typically involve the loose, sandy, colluvial deposits derived from the glacial and nonglacial sediments that form the bluffs [Galster and Laprade, 1991].

[29] The instrumented hillslope is a steep (45°) coastal bluff covered by a thin, sandy (<2.0 m) colluvium, and the surface morphology is generally planar (Figure 8b). The surficial materials on the bluff were initially instrumented with water content instruments and open-tube piezometers equipped with pressure transducers and an automated data acquisition system to collect hourly readings in September of 2001 [Baum et al., 2005]. However, no positive pore water pressures were observed during 2 years of operation and the piezometers were abandoned in September of 2003. In October of 2003, two water content profilers and two nests of 6 tensiometers were installed. During the 16 month period this instrument array was operational, no positive pore water pressures or volumetric water contents in excess of 0.40 were recorded in the hourly data [Godt et al., 2009]. Measurements of the bulk density performed in both the lab and field indicate that the saturated volumetric water content or porosity of the colluvium is about 0.40.

[30] On 14 January 2006 a 25 m long by 11 m wide shallow landslide initiated in colluvium near the instrument array [Godt et al., 2009]. The depth of the failure surface was between 1.0 and 2.0 m below and roughly parallel to the original ground surface. The failure surface was apparently coincident with the contact between the loose sandy colluvium and the underlying, better consolidated glacial outwash sand (Figure 8c). The slide exposed a thin (<2 m) silt bed near the headscarp and a shallow zone (<0.8 m below the ground surface) of small diameter (<2 mm) blackberry and grass roots. Only a few larger diameter,
partially decayed Alder tree roots penetrated the failure surface. The landslide deposit did not mobilize as a debris flow [Godt et al., 2009].

The contributing area \( A \) above the site is between about 32 and 64 m\(^2\) estimated from a 1.86 m (6 ft) digital elevation model derived from airborne laser swath mapping. Taking a reference soil water diffusivity \( D_0 \) of \( 1 \times 10^{-4} \) m\(^2\) s\(^{-1}\) [Baum et al., 2010], the characteristic times for slope-normal and slope-parallel pressure diffusion [Iverson, 2000] are 1.4–5.4 and 89–178 h, respectively.

### 6.1. Numerical Modeling of Transient Flow

We applied a numerical, one-dimensional, finite element solution [Simůnek et al., 2008a, 2008b] to equation (9a) to simulate the pore water response to rainfall at the Edmonds site. The numerical solution allows us to take advantage of available inverse parameter estimation routines, and a more accurate description of the soil water characteristics of the hillside materials and boundary conditions of the field setting. The relation between pressure head \( \psi \) and volumetric water content \( \theta \) is given by the van Genuchten [1980] formulation

\[
\theta(\psi) = \theta_s - \frac{\theta_s - \theta_r}{[1 + (\alpha \psi) \omega]^n}
\]

where \( \theta_r \) and \( \theta_s \) are the residual and saturated volumetric water contents, respectively, and \( \alpha \), \( n \), and \( \omega \) are curve fitting parameters. The parameter \( \alpha \) is considered to be the approximate inverse of the air entry pressure head [van Genuchten, 1980]. The pressure head-dependent hydraulic conductivity \( K(\psi) \) is predicted using the statistical pore size distribution model of Mualem [1976]:

\[
K(\psi) = K_s S_e^{0.5} \left[ 1 - (1 - S_e^{0.5})^2 \right]
\]

where \( S_e \) is the effective saturation.

The numerical solution allows us to use a split-sample, inverse-modeling approach [Klemeš, 1986] described in section 6.2 to estimate the soil water characteristics from field monitoring information. The model domain was a one-dimensional 3.76 m deep homogeneous profile above a no-flow boundary. The infiltration flux at the upper boundary was taken from measured rainfall (Figure 9a). Initial conditions were assumed to be hydrostatic and in equilibrium with a water table coincident with the lower boundary.

Figure 5. Profiles of (a) pressure head, (b) effective saturation, (c) suction stress, and (d) factor of safety for medium sand with \( \alpha = 1.0 \), saturated hydraulic conductivity \( K_s = 7.5 \times 10^{-6} \) m s\(^{-1}\), and material strength properties in Table 1. Prescribed flux at the ground surface is equivalent to \( K_s \).
and consistent with the measured pressure head profile in the middle of September 2005 (Figure 9c).

6.2. Comparison of Model Results With Monitoring Observations

[33] Soil water characteristics were estimated using an inverse modeling procedure that systematically minimized the differences between observed and modeled pressure heads and volumetric water contents [Hopmans et al., 2002; Šimůnek et al., 2008a]. The differences are expressed using an objective function [Hopmans et al., 2002] and minimized with the Levenberg-Marquardt optimization algorithm [Marquardt, 1963; Šimůnek et al., 2008a]. Measured rainfall (Figure 9a), volumetric water contents (Figure 9b), and pressure heads (Figure 9c) at a depth of 1.5 m from the downslope monitoring array (Figure 8d) for the period from 28 October 2005 to 21 November 2005 were used to inversely estimate the saturated hydraulic conductivity, saturated and residual volumetric water contents, and the fitting parameters for the van Genuchten [1980] form of the soil water characteristic (Table 3). The choice of the October–November period ensured that we obtained soil hydraulic parameters that represent wetting conditions, which is likely the case when landslides are initiated [Ebel et al., 2010]. Table 4 provides the root-mean-square error (RMSE) for each of the simulated and derived results for the forward modeling period after 21 November 2005.

[35] Figure 9b compares observed and volumetric water contents simulated using the numerical model. The agreement between observations and model results between 28 October 2005 and 21 November 2005 is best at the 1.5 m depth as this was the target for the inverse modeling procedure. At both the 1.0 and 1.5 m depths, the timing of changes in volumetric water content match quite well (Table 4), although in general, modeled volumetric water contents decrease more rapidly following the cessation of rainfall. This may result because we ignore any lateral downslope flow or hysteresis in the soil water characteristics in the one-dimensional model, both of which would tend to attenuate any decrease in volumetric water content during drainage. Comparison of the contributing area and landslide failure depths at the site indicates that 1-D analysis of infiltration over the time scale of typical rainstorms that initiate landslides at this location are likely of sufficient accuracy [Godt et al., 2006], but that longer-term simulations (e.g., seasonal) may be improved by considering downslope redistribution of soil water. However, the direction of flow in the unsaturated zone will tend to be vertical or normal to the slope surface as the soil wets and only downslope under drying conditions [e.g., Lu et al., 2010].

[36] Compared to the 1.5 m depth, the agreement between the modeled and observed volumetric water contents is not as close at 1.0 m depth. Initially the simulated volumetric water content is about 0.08 too large compared to the observations. This apparent bias toward higher

![Figure 6. Profiles of (a) pressure head, (b) effective saturation, (c) suction stress, and (d) factor of safety for fine sand with $c_s = 0.7$, saturated hydraulic conductivity $K_s = 5 \times 10^{-2}$ m s$^{-1}$, and material strength properties in Table 1. Prescribed flux at the ground surface is equivalent to $K_s$.](image-url)
volumetric water contents at the 1.0 m depth is evident throughout the simulation (Figure 9b). This discrepancy likely results from minor differences in the hillside materials and thus differences in soil water characteristics at the two depths.

The timing and magnitude of the modeled and observed pressure head variation are in general agreement at both the 1.0 and 1.5 m depths (Figure 9c). The arrival of the wetting front at the 1.0 m depth in the end of October 2005 is well captured by the simulation. The simulated arrival of the wetting front at 1.5 m depth is less abrupt and delayed compared to the observation (Figure 9c). However, differences between observed and modeled pressure head decrease as the soil becomes wetter. Modeled pressure heads are greater than the observation at the 1.0 m depth after the beginning of November and greater at both depths during the wet period leading to the landslide at the site beginning in the middle of December 2005 (Figure 9c). Modeled pressure heads also show a greater response to rainfall than observations during the period from the end of December to 14 January. This may result from limitations of pressure transducers and tensiometers to resolve pressure heads near zero.

Suction stress was calculated using equation 2 and is shown for both the modeled and observed volumetric water contents and pressure heads assuming a saturated volumetric water content of 0.4 (Figure 9d). At the 1.0 m depth, suction stress calculated from the monitoring data increases from about −12 kPa to −4 kPa after the rainy period in late October 2005 and fluctuates around −5 kPa for the remaining period (Figures 9a and 9d). At 1.5 m the calculated suction stress has a similar pattern to that calculated for the 1.0 m depth, but the increase from about −18 kPa to about −4 kPa follows the increase at 1.0 m by 2.6 days as the wetting front moves through the soil (Figure 9c) and remains at about −4 kPa.

The differences in timing between modeled changes in suction stress compared to those calculated from the monitoring data mimic the differences between the modeled and observed pressure heads (Figures 9d and 9c). The differences between the modeled and calculated magnitudes are generally less than a few kilopascals, but have a substantial impact on modeled factors of safety (Table 4).

Factors of safety calculated from the monitoring data are greater than 1.5 at both 1.0 and 1.5 m depth prior to the onset of rainfall and consequent increases in volumetric water content, pressure head, and suction stress that occur in the beginning of November 2005 (Figure 9).
Factors of safety from the monitoring data at 1.5 m decrease to near unity (indicating the potential for slope failure) at this time and remain close to 1.0 for the remainder of the record. At the 1.0 m depth, factors of safety reach a minimum of about 1.2 during several periods at the beginning of November, December, and at the end of December.

On 14 January 2006, factors of safety calculated from the monitoring data were less than unity at the 1.5 m depth for about 50 h prior to the occurrence of the landslide [Godt et al., 2009].

Factors of safety calculated from the volumetric water contents and pressure heads obtained from the monitoring data at 1.5 m decrease to near unity (indicating the potential for slope failure) at this time and remain close to 1.0 for the remainder of the record. At the 1.0 m depth, factors of safety reach a minimum of about 1.2 during several periods at the beginning of November, December, and at the end of December. On 14 January 2006, factors of safety calculated from the monitoring data were less than unity at the 1.5 m depth for about 50 h prior to the occurrence of the landslide [Godt et al., 2009].
numerical model are greater than unity at both the 1.0 and 1.5 m depths in September and much of October 2005. They decrease at both depths in early November in response to the rainfall, increases in volumetric water content, pressure head, and suction stress (Figure 9). In late December the modeled factors of safety at both depths approach unity and indicate potential instability beginning on 26 December and remain there for 72 h. After this time, factors of safety at the 1.0 m depth increase to more than 1.0, but those at 1.5 m remain less than 1.0 through the end of the period. Figure 10 highlights factors of safety for the period from late December 2005 until the onset of the landslide that damaged the instrumentation on 14 January at the two depths calculated from both the monitoring data and model results. Factors of safety from the modeling results at the 1.5 m depth fluctuates around 0.9 for much of the period prior to 14 January 2006 in a similar manner as to those from the field data, but are slightly smaller. However, factors of safety from the model results at the 1.0 m depth are greater than unity, although not as large as those from the field data, for the period (Figure 10). On 14 January 2006 factors of safety at 1.0 m calculated from the monitoring data decrease to 1.0 seven hours prior to landslide occurrence.

7. Concluding Discussion

[42] We have extended a framework for infinite slope stability analysis for variably saturated materials [Lu and Godt, 2008] from steady state to transient conditions to assess the conditions leading to landslide occurrence above the water table. Four hypothetical hillslopes (comprising coarse, medium and fine sand, and silt) were examined for their representative range of materials for which shallow landslides may occur under partially saturated conditions. Results show that the depth and timing of the onset of potential slope instability are dependent on the soil water characteristics of the hillside materials. Given the same soil strength, the failure depth in the coarse sand hillslope is a few tens of centimeters below the ground surface compared to about a meter for the fine sand example. This results in part from the differences in the contribution of suction stress to stability. For the limiting case of rainfall flux at the ground surface equivalent to the saturated hydraulic conductivity of the soil, and for hydrostatic initial conditions, sand hillslopes can be destabilized in a few hours whereas in contrast, silt hillslopes are destabilized in tens of days. Less thick deposits, permeability contrasts, accreting water tables, and wetter initial conditions all tend to reduce the time needed to achieve potential instability in the unsaturated zone under a given flux of water at the ground surface. Given the lower boundary conditions of a fixed water table, rainfall flux much less than the saturated hydraulic conductivity will generally not provoke instability [Lu and Godt, 2008].

[43] The case study from the instrumented hillslope along the Puget Sound is also presented to test the framework for assessing the transient conditions leading to variably saturated landslide occurrence. An inverse modeling approach is used to estimate the soil water characteristics from field monitoring data, although improvements in the prediction of landslide timing and thickness might be achieved using other inverse modeling algorithms [e.g., Wöhling et al., 2008]. The comparison between model results and field observations implies that the model is suitable for slope stability assessment under transient seepage conditions. The model is also able to predict approximate depth and timing for possible occurrence of landslides. However, practical application of this approach would likely require interpretation of results in a probabilistic manner.

References


