DISCUSSIONS AND CLOSURES

Discussion of “Using a Small Ring and a Fall-Cone to Determine the Plastic Limit” by Tao-Wei Feng

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J. Kodikara1; H. N. Seneviratne2; and C. V. Wijayakulasooriya3
1Senior Lecturer, Dept. of Civil Engineering, Monash Univ., Building 60, Wellington Rd., Victoria 3800, Australia.
2Professor, Dept. of Civil Engineering, Univ. of Peradeniya, Sri Lanka.
3Principal Engineer, Dept. of MainRoads, Queensland, Australia.

The author has presented a laboratory test that uses a fall-cone to determine the plastic limit of fine-grained soils as an alternative to the traditional thread-rolling test. The traditional method is more operator-dependent, and those who are new to soil engineering, such as undergraduate students, may consider it to be a low-tech test. This view has potential to devalue the discipline as a whole and shy away good workers in the future. Therefore, attempts to find new, yet simple and reliable, tests to determine plastic limit must be encouraged.

The method proposed by the author, following from his previous work (Feng 2000, 2001), draws from the original idea proposed by Wroth and Wood (1978), who made the following assumptions:

1. Using the original concepts of critical state, the relationship between w and ln(cu) is linear, where w=moisture content and cu=undrained shear strength.
2. (cu d2/W)=constant for a particular cone geometry, where d=depth of cone penetration and W=cone weight.
3. The undrained shear strength increases by 100-fold from liquid limit to plastic limit.

Assumptions (1) and (2) lead to a linear relationship between w and ln(d), Feng (2000, 2001, as well as the paper under discussion) has retained Assumptions (2) and (3) but has shown that Assumption (1) is generally not valid. Instead, relationships of ln(w) with ln(cu), hence, relationships of ln(d) with ln(w), were shown to be linear for all soils tested; and this linearity is herein referred to as Assumption (1a). Therefore, the accuracy of Feng’s method depends on the validity of Assumptions (1a), (2), and (3).

On the basis of published material and their test results, Kodikara et al. (1986) also disputed the general validity of Assumption (1) and argued that it could be replaced with Assumption (1a). The 13 soil types used in their testing were obtained from various parts of Sri Lanka, covering a range of tropical climatic and geological conditions, as well as one sample from the United States. Using Assumptions (1a), (2), and (3), Kodikara et al. proposed two methods to determine the plastic limit from the liquid limit fall-cone test, BS 1377 (1975).

Method (1) involves using two cones of the same geometry but different weights (W1=230 g and W2=80 g) and uses Assumptions (1), (2), and (3). Assumption (1) can be written as

\[ \ln w = -a_1 \ln cu + a_2 \] (1)

where \( a_1 \) and \( a_2 \) are constants. Assumption (2) gives

\[ \frac{cu d^2}{W} = a_3 \] (2)

where \( cu \) = undrained shear strength; \( d \) = cone penetration; and \( W \) = cone weight. Using Eq. (2) in Eq. (1) gives

\[ \ln w = 2a_1 \ln d - a_1 \ln(a_1 W) + a_2 \] (3)

Eq. (3) shows that the relationship between \( \ln w \) and \( \ln d \) is linear [i.e., Assumption (1a)], for a given cone weight of \( W \). If two experiments are conducted with different cone weights \( W_1 \) and \( W_2 \), it can be easily shown from Eq. (3) that the two relations of \( \ln w \) versus \( \ln d \) are separated by a vertical distance of \( Y \), as shown in Fig. 1, where \( Y = a_1 \ln(W_1/W_2) \). Then using this relationship for \( Y \) with Assumption (3), the plastic limit (\( w_{pl} \)) in Method 1 is determined by

\[ w_{pl} = \frac{w_{ll}}{100^a} \] (4)

where \( a = Y/\ln(W_1/W_2) \).

Method 2 is conceptually the same as Feng’s proposed method and involves extending the points obtained for liquid limit back-
ward to obtain \( w_{PL} \) corresponding to \( d = 2 \text{ mm} \) [or \( \ln(d) = 0.693 \)], as shown in Fig. 2. The penetration depth of 2 mm arises from Assumptions (2) and (3) and the use of \( d = 20 \text{ mm} \) for the liquid limit determination.

Table 1 shows the results for 14 soil types. As Feng has found, the method of Wroth and Wood is unsatisfactory for the soils tested. Methods 1 and 2 provide reasonably close results, as shown in Figs. 3 and 4, but underestimate PI or overestimate \( w_{PL} \) from the thread method by 12 and 18\%, respectively. Feng’s test method, however, appears to provide closer agreement with the thread-rolling method. The major differences between Method 2 and the method used in the paper are that Feng has used a smaller ring and has used penetration depths between 3 and 10 mm, whereas Kodikara et al. (1986) uses penetration depths between 10 and 30 mm, as suitable for the liquid limit test. The method used in the paper, despite being less accurate, predicts the plastic limit from the liquid limit test results directly. In addition, the author has not described the difficulty of working with a relatively large amount of soil close to the plastic limit.

The results presented in this discussion highlight the validity of a log-log linear relation [i.e., Assumption (1o)]. More recent thermomechanical research (e.g., Collins and Kelly 2003) also supports a log-log linear relationship. Assumption (2) is based on dimensional analysis and can be considered reasonably accurate, provided that the influencing factors are kept controlled (e.g., cone surface friction and cone angle). As the author has pointed out, the plastic limit computed is not very sensitive to the 100-fold strength ratio [Assumption (3)] and may be considered as a benchmark value. Finally, although the test method presented appears to be a viable alternative to the thread method, it should not deter people from researching other more suitable methods, capturing the fundamental behavior of fine-grained soils.

**References**


Feng, T. W. (2001). “A linear log \( d \)-log \( w \) model for the determination of

**Table 1. Test Results for Various Soils from Sri Lanka [Adapted from Kodikara et al. (1986)]**

<table>
<thead>
<tr>
<th>Sample location</th>
<th>Liquid limit</th>
<th>Thread method</th>
<th>Wroth and Wood (1978)</th>
<th>Method 1 (using two cone weights, ( W_1 = 230 \text{ g} ) and ( W_2 = 80 \text{ g} ))</th>
<th>Method 2 (plastic limit corresponding to ( d = 2 \text{ mm} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vavunia</td>
<td>44.3</td>
<td>25.2</td>
<td>36.3</td>
<td>28.1</td>
<td>24.3</td>
</tr>
<tr>
<td>Matale</td>
<td>35.2</td>
<td>16.2</td>
<td>23.4</td>
<td>17.5</td>
<td>17.6</td>
</tr>
<tr>
<td>Samanalawewa</td>
<td>45.4</td>
<td>18.3</td>
<td>24.8</td>
<td>21.3</td>
<td>19.1</td>
</tr>
<tr>
<td>Kagalle</td>
<td>58.3</td>
<td>25.5</td>
<td>32.6</td>
<td>26.3</td>
<td>26.0</td>
</tr>
<tr>
<td>Badulla</td>
<td>77.4</td>
<td>47.1</td>
<td>76.0</td>
<td>52.8</td>
<td>51.4</td>
</tr>
<tr>
<td>Uda Walawe</td>
<td>40.1</td>
<td>22.6</td>
<td>30.6</td>
<td>24.5</td>
<td>22.0</td>
</tr>
<tr>
<td>Aranayake</td>
<td>52.1</td>
<td>24.6</td>
<td>31.1</td>
<td>30.0</td>
<td>25.2</td>
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<tr>
<td>Tellipalai</td>
<td>74.4</td>
<td>43.1</td>
<td>71.0</td>
<td>48.9</td>
<td>49.1</td>
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<tr>
<td>Kalanitissa</td>
<td>81.1</td>
<td>49.0</td>
<td>122.6</td>
<td>59.7</td>
<td>55.7</td>
</tr>
<tr>
<td>Kandy</td>
<td>93.2</td>
<td>45.5</td>
<td>86.7</td>
<td>60.0</td>
<td>54.9</td>
</tr>
<tr>
<td>Borelesgamuwa</td>
<td>83.2</td>
<td>36.4</td>
<td>65.1</td>
<td>47.5</td>
<td>47.8</td>
</tr>
<tr>
<td>Kopay</td>
<td>24.9</td>
<td>10.1</td>
<td>23.4</td>
<td>16.1</td>
<td>15.8</td>
</tr>
<tr>
<td>Maduru-Oya</td>
<td>25.3</td>
<td>-</td>
<td>18.2</td>
<td>13.6</td>
<td>13.7</td>
</tr>
<tr>
<td>Illite (USA)</td>
<td>67.7</td>
<td>35.3</td>
<td>47.3</td>
<td>35.7</td>
<td>34.4</td>
</tr>
</tbody>
</table>

![Fig. 3. Comparison between Method 1 and thread method [adapted from Kodikara et al. (1986)]](image3)

![Fig. 4. Comparison between Method 2 and thread method [adapted from Kodikara et al. (1986)]](image4)
The fall-cone method proposed by the writer includes using a small specimen ring as a necessity to run a good test. The major reason is that, as described in the paper, the amount of soil required for one plastic limit fall-cone test is therefore greatly reduced. The small specimen ring takes only a 6.3 cm³ soil sample, as compared with a 93 cm³ soil sample taken by the standard specimen cup used for the fall-cone liquid limit test. Another important reason is that the specimen ring facilitates preparing good-quality specimens (Feng 2000). In those plastic limit fall-cone tests carried out by the writer, the lowest depth of penetration was around 4 mm, at which point the soil paste was still sufficiently plastic that preparing a specimen with the small ring was rather easy.

The discussers’ Method 1 may be of little practical significance, since it is neither convenient nor accurate as compared with their Method 2. In fact, the discussers’ Method 1 is likely to frighten practicing engineers since it requires a double amount of work for estimating the capillary rise velocity. The discussers’ Method 2 is compared with the writer’s Method 2. In Table 1, Method 2 is applied to a wide range of water content. How-

tor with the water content. Table 1 indicates that Kodikara’s method includes an extrapolation of the measured linear regression log d − log w relationship from at least 15 to 2 mm to estimate the plastic limit. Such an extent of extrapolation may not be a comfortable practice for most engineers. It is therefore logical to obtain the linear log w − log d relationship as close to the plastic limit as possible to better estimate the plastic limit. This consideration is the primary one in the fall-cone plastic limit method proposed by the writer. Finally, the writer agrees that new methods for determining the plastic limit are always welcome and worthy of research by creative civil engineers.

Table 1. Comparisons Between Kodikara’s Method and Feng’s Method

<table>
<thead>
<tr>
<th>Item</th>
<th>Kodikara’s method</th>
<th>Feng’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen container</td>
<td>Cup</td>
<td>Ring</td>
</tr>
<tr>
<td>Assumptions</td>
<td>(1a), (2), (3)</td>
<td>(2), (3)</td>
</tr>
<tr>
<td>Volume of specimen</td>
<td>93 cm³</td>
<td>6.3 cm³</td>
</tr>
<tr>
<td>Minimum range of extrapolation</td>
<td>Between 15 and 2 mm</td>
<td>Between 4 and 2 mm</td>
</tr>
</tbody>
</table>

References


Discussion of “Rate of Capillary Rise in Soil” by N. Lu and W. J. Likos

The authors have presented an interesting analytical framework for estimating the capillary rise rate in unsaturated soils, considering the variation of soil unsaturated hydraulic conductivity with respect to suction head. The authors assumed the same hypotheses as those assumed originally by Terzaghi (1943) which are as follows: (1) a hydraulic gradient related to capillary rise defined by Eq. (1); and (2) Darcy’s law describing capillary rise is valid [Eq. (2)]. In Eq. (2), the term dz/dt denotes the seepage velocity. On the basis of Lambe and Whitman (1979), defining the seepage velocity using Eq. (2) implicitly involves three more assumptions that the authors do not address. Assumptions are as follows:

- The water path within the soil in a macroscopic scale is a straight line.
- The seepage velocity magnitude evaluated by using Eq. (2) is indeed the average seepage velocity developed through the interconnected soil pores (of different sizes) conducting water.
- The seepage velocity (dz/dt) is equal to the discharge velocity (q) divided by soil porosity (n).

Concerning the first implicit assumption, it is well known that the water path through soil pores is tortuous. Burdine (1952) and Corey (1954) observed that the water path tortuosity increases with the soil’s degree of saturation decrease. Assuming a linear water path automatically implies that the effect of water-path tortuosity on the capillary rise rate is disregarded. The authors did not discuss the effect of assuming a straight-line water path during capillary rise independent of the soil’s degree of saturation. Corey (1994) presents a simple model that describes the effect of...
The influence of the second and third implicit assumptions on the analytical model that the authors proposed is addressed as follows. Once the authors assumed that the soil hydraulic conductivity was a function of suction head, the second assumption and mainly the third implicit assumption \((dz/dt=q/n)\) should be carefull analyzed, mostly considering that the soil starts undergoing unsaturated flow. The suction head magnitudes reached during unsaturated flow can be higher than the soil air entry-level magnitude. Fig. 1 shows a theoretical model that describes the water flow through an unsaturated soil control volume. Fig. 1(b) shows the tortuosity of the water path in a microscopic scale and the straight-line water path assumed on a macroscopic scale, whereas Fig. 1(c) depicts the theoretical seepage velocity model describing the volume of solids \((V_s)\), the volume of water stored within the control volume \((V_w)\), and the volume of voids fulfilled with air \((V_a)\). The model shown in Fig. 1(c) is similar to the model proposed by Lambe and Whitman (1979) in that it considers an average seepage velocity denoted by the lump sum of the cross-section areas of the voids filled with water \((A_v)\) and the water path in a macroscopic scale as a straight line [see Fig. 1(b)]. However, the model presented in Fig. 1(c) differs from the model proposed by Lambe and Whitman (1979), since it assumes that the air phase within the soil is continuous (denoted by the volume of air, \(V_a\)). This assumption indirectly implies that the suction head within the soil control volume is higher than the soil air entry-level magnitude. On the basis of the principle of continuity, the seepage velocity within the unsaturated soil control volume equals

\[
\frac{dz}{dt} = \frac{q}{A_w} A
\]

where \(A\) = cross-section area of the control volume and \(A_w\) = lump sum of the cross-section areas of the pores conducting water. Multiplying the numerator and the denominator of the term on the right side of Eq. (1) by the control volume length \((L)\) gives

\[
\frac{dz}{dt} = \frac{q V}{V_w}
\]

where \(V\) = volume of the control volume and \(V_w\) = volume of water stored within the control volume since the soil volumetric water content is equal to the ratio between the volume of water stored within the control volume and the volume of the control volume \((V_w/V)\), Eq. (2) becomes

\[
\frac{dz}{dt} = \frac{q}{\theta}
\]

where \(\theta\) = soil volumetric water content. Eq. (3) indicates that the seepage velocity within an unsaturated soil is a function of the soil volumetric water content. Analysis of Eq. (3) indicates that the seepage velocity within an unsaturated soil is a function of soil porosity only for suction-head magnitudes lower than the soil air entry level. For suction-head magnitudes higher than the air entry-level magnitude, the seepage velocity developed through an unsaturated soil is a function of the volumetric water content at the respective soil cross section undergoing flow. The assumption of seepage velocity as a function of soil porosity independently of the soil’s degree of saturation (as proposed by the authors) leads to consistent underestimation of the actual seepage velocity developed during capillary rise. Considering the seepage velocity as defined by Eq. (3), it can be concluded that the governing equation for the rate of capillary rise proposed by the authors in their Eq. (5) might be better defined as

\[
\frac{dz}{dt} = \frac{k_s \exp(-\alpha z)}{\theta} \frac{h_c - z}{z}
\]

Since the volumetric water content is also a function of the suction head (defined by the soil-water retention curve), the solution of Eq. (4) no longer can be conducted following the steps, proposed by the authors [Eq. (10) in the Appendix]. The analytical solution of Eq. (4) probably provides a better fit to the experimental data of Lane and Washburn (1946).

References


Water Resources, Highlands Ranch, Colo.

The authors have presented an analytical solution for water capillary rise in porous media. The solution allows for the reduction in hydraulic conductivity resulting from desaturation. Unfortunately, the authors have not considered the water retention effects of desaturation in their analysis, despite showing this effect schematically in their Fig. 1. Their solution is therefore inaccurate for capillary rise prediction. In their analysis, Eq. (2) represents the upward rate of capillary flux, which was calculated by assuming that the medium is saturated. They have used the same equation when they incorporated the reduction of hydraulic conductivity attributable to desaturation, given in their Eq. (4). Allowing for desaturation, the authors’ Eq. (2) should be written as

\[ q = n S \frac{dz}{dt} = k_e \exp(-\alpha h) i \]  

(1)

The degree of saturation \( S \) is a function of matric suction head \( h \) (i.e., the water-retention function or the soil water characteristic curve); and for the sake of this analysis, if we approximate it to follow the same function as the hydraulic conductivity function, Eq. (1) can be rewritten as

\[ q = n \exp(-\alpha h) \frac{dz}{dt} = k_e \exp(-\alpha h) i \]  

(2)

Eq. (1) reduces to the authors’ original Eq. (2) applicable to saturated conditions. In other words, if we assume that the soil characteristic curve and the hydraulic conductivity follow the same functional form, the authors’ Eq. (3) [derived by Terzaghi (1943), as cited by the authors] is applicable under both saturated and desaturated conditions. This outcome is somewhat intriguing, but it means that the reduction of hydraulic conductivity is compensated by the reduction in pore space filled by the capillary rise, thereby keeping the rate of rise pretty much the same. Although the authors’ Eq. (3) is simple, the actual capillary rise process may deviate from it because of a number of other complications. As noted by Lee et al. (2003), the derivation of the authors’ Eq. (3) assumes a sharp wetting front, steady-state conditions after initial filling, and the soil water characteristic curve applicable to wetting. In addition, inertial effects can play a significant role at the beginning of rise, causing the initial behavior to deviate from the theory. The functional form of the soil water characteristic curve can significantly differ from the hydraulic conductivity function, especially for a wetting phase. The applicable value of \( h_i \) in the authors’ Eq. (3) is actually an average value for the pore matrix. If the assumption of sharp wetting front is valid, then it may be assumed that \( h_i \) is a constant. But under general desaturated conditions, applicable \( h_i \) can vary, as the capillary rise progresses. Another aspect to consider is that after reaching the rise relevant to the residual water content of wetting, the vapor flux will dominate the capillary flux. The exact analysis of capillary rise may be complicated and will depend heavily on material properties and structure. Nevertheless, simplified forms are still useful, particularly for some materials and field applications, provided that their region of validity is properly examined and stated (e.g., Lee et al. 2003). The authors’ analytical solution given in Eq. (6), however, is not suitable for capillary rise or rate prediction under desaturated conditions because it fails to properly account for desaturation.

References


Closure to “Rate of Capillary Rise in Soil” by N. Lu and W. J. Likos

Ning Lu\(^1\) and William J. Likos\(^2\)

\(^1\)Professor, Colorado School of Mines, Div. of Engineering, Golden, CO 80401. E-mail: ninglu@mines.edu

\(^2\)Assistant Professor, Univ. of Missouri–Columbia, Columbia, MO 65211. E-mail: likosw@missouri.edu

The writers appreciate the discussers’ interest in the paper. The discusser Kodikara provides four suggestions to improve our analytical solution for the rate of capillary rise in soil: (1) explicit consideration of pore desaturation above the air entry head; (2) consideration of inertial effects; (3) redefinition of the ultimate height of capillary rise, \( h_r \); and (4) consideration of vapor flux. The discusser Dell’ Avanzi suggests explicit consideration of water-path tortuosity and echoes the first discusser’s comments regarding pore desaturation above the air entry head.

Both discussers are correct in pointing out that the pore space in soil above the capillary fringe (air entry head) is partially saturated during capillary rise and that the authors’ analytical solution for the rate of rise does not explicitly consider this effect. Explicit consideration of desaturation, however, may not be amenable to an improved analytical solution. Kodikara offers one approach for doing so by considering a soil-water characteristic curve (SWCC) with the same functional form as the hydraulic conductivity function (HCF) adopted by the authors (Gardner 1958). This approach is shown to lead to Terzaghi’s (1943) original solution for the rate of capillary rise [Eq. (3) of the paper], which was derived under the assumption of saturated conditions.

The authors consider two deficiencies with the discussers’ proposed approach. First, Terzaghi’s solution, as widely recognized by many researchers and as illustrated by comparison with experimental data (Figs. 3 and 4 of the writers’ paper) significantly
deviates from observed behavior, typically by several orders of magnitude. Second, although the discusser presents an elegant mathematical way to recover Terzaghi’s solution, assuming that the SWCC follows the same functional form and same exponential decay constant $\alpha$ as the HCF is not physically representative for most soils. The discusser recognizes this deficiency by stating that “the functional form of the SWCC can significantly differ from the HCF,” which renders the proposed approach by the discusser somewhat contradictory.

Dell’ Avanzi suggests a governing equation for the rate of rise that incorporates desaturation through the SWCC, and he asserts that its solution will provide a better fit to experimental data, but he, unfortunately, does not present the solution or an approximation to it. Without actually presenting a new solution and comparing it with previous solutions and experimental data, the discusser’s conclusion is speculative and remains unjustified. Dell’ Avanzi also suggests that incorporating Corey’s (1994) tortuosity model into the authors’ analysis may render an improved analytical solution. Corey’s model describes water-path tortuosity as a function of soil degree of saturation as

$$T(S) \approx T_{\text{sat}} \left( \frac{1 - S_r}{S - S_r} \right)^2$$

where $S_r =$ residual degree of saturation and $T_{\text{sat}} =$ saturated tortuosity, defined as

$$T_{\text{sat}} = \left( \frac{l}{l_i} \right)^2$$

where $l =$ macroscopic distance and $l_i =$ actual fluid particle travel distance. Conceptually, the authors agree that tortuosity will slow down the rate of capillary rise and that the magnitude of tortuosity generally increases as the soil’s degree of saturation decreases, as illustrated by the discusser’s analysis. However, information indicating how Corey’s model or similar models could be incorporated into the authors’ analytical framework and whether such an effort is amenable to analytical solution is not provided by the discusser. Consequently, the discusser’s suggestion for improving the authors’ analysis remains incomplete.

Inertial effects are trivial with respect to the magnitude of the capillary head $h_c$, which may be reasoned quantitatively by considering Lane and Washburn’s (1946) experimental data (Figs. 3 and 4 of the writers’ paper). From Bernoulli’s well-known equation, the inertial head can be expressed as $h_i = \frac{v^2}{2g}$ ($v =$ seepage velocity and $g =$ gravity constant), which from the early rise time data are on the order of $10^{-8}$ to $10^{-9}$ m. The typical capillary head $h_c$, however, is on the order of decimeters and meters. Theoretical and experimental analyses of capillary rise phenomena in idealized capillary tubes have indicated that inertial effects may indeed be important at the initiation of capillary rise. However, the temporal and spatial scales bounding the inertial regime [rise time less than about 1 s and rise height less than about 1 cm (e.g., Quéré 1997)] are insignificant with respect to the much longer and much larger scale of practical capillary rise problems in soil.

Regarding the definition of the capillary head $h_c$, Kodikara considers it as a variable that is only uniquely defined under the assumption of a sharp wetting front. The authors disagree. Similar to the SWCC, the capillary head $h_c$ is characteristically unique for each given soil; physically, for any given soil column at a given structure, no matter how complex the pore fabric and corresponding diffusivity of the wetting front, there will be one ultimate height of capillary rise and one water content distribution at equilibrium (i.e., the SWCC). However, the available driving head, as conceptualized in Eq. (1) of the writers’ paper, varies not only as a function of time (position of the wetting front) but also in the way that the position of wetting front is defined (i.e., sharp versus diffusive).

Kodikara suggests that when capillary rise approaches steady state, vapor flux could dominate upward water transport. The writers agree. Vapor transport, however, is an entirely different physical process which occurs primarily in a different domain (near soil-atmosphere interface) than that considered in the writers’ transient analytical solution for the liquid phase in the soil domain.

In summary, although capillary rise in soil certainly involves additional complexities not embedded in the authors’ analysis, our theoretical model and its analytical solution [Eq. (6) of the writers’ paper], as supported by comparison with experimental data, provides significant improvement over the previous analytical prediction [Eq. (3) of the paper] for the rate of capillary rise.

References


