Suction Stress Characteristic Curve for Unsaturated Soil

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Abstract: The concept of the suction stress characteristic curve (SSCC) for unsaturated soil is presented. Particle-scale equilibrium analyses are employed to distinguish three types of interparticle forces: (1) active forces transmitted through the soil grains; (2) active forces at or near interparticle contacts; and (3) passive, or counterbalancing, forces at or near interparticle contacts. It is proposed that the second type of force, which includes physicochemical forces, cementation forces, surface tension forces, and the force arising from negative pore-water pressure, may be conceptually combined into a macroscopic stress called suction stress. Suction stress characteristically depends on degree of saturation, water content, or matric suction through the SSCC, thus paralleling well-established concepts of the soil–water characteristic curve and hydraulic conductivity function for unsaturated soils. The existence and behavior of the SSCC are experimentally validated by considering unsaturated shear strength data for a variety of soil types in the literature. Its characteristic nature and a methodology for its determination are demonstrated. The experimental evidence shows that both Mohr–Coulomb failure and critical state failure can be well represented by the SSCC concept. The SSCC provides a potentially simple and practical way to describe the state of stress in unsaturated soil.

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Introduction

In 1943, Karl Terzaghi defined effective stress as stress representing "that part of the total stress which produces measurable effects such as compaction or an increase of the shearing resistance" (Terzaghi 1943). In mathematical form, Terzaghi's effective stress σ' is expressed as the difference between total stress σ and pore-water pressure u_w (Terzaghi 1936)

$$\sigma' = \sigma - u_w \tag{1}$$

If, on the other hand, the compressibility of the grains c_s is a considerable fraction of the compressibility of the granular skeleton c (as in most rocks or engineering materials such as concrete) Skempton (1960) showed that effective stress is better expressed as

$$\sigma' = \sigma - \left(1 - \frac{c_s}{c}\right)u_w \tag{2}$$

In developing Eq. (2), Skempton clarified that "Terzaghi's equation does not give the true effective stress, but gives an excellent approximation for the case of saturated soils" (Mitchell 1976). In other words, for most soils, the compressibility ratio term in Eq. (2) can be ignored and Eq. (2) is reduced to Eq. (1). The deformation and strength of soil in response to a change in stress conditions depends almost exclusively on the corresponding change in effective stress defined by Eq. (1). As such, Terzaghi's effective stress equation has proven to be extremely useful in practical engineering applications and has without question become the cornerstone for stress, strain, and strength considerations in current geotechnical engineering practice.

What, however, are the forces in soil that are not explicitly included in the effective stress defined by Eq. (1) but do indeed produce measurable effects on deformation and strength? Such forces exist in both saturated and unsaturated soils. Classical studies on saturated soils, for example, have demonstrated that the missing forces are physicochemical in nature, namely van der Waals attraction and electrical double-layer repulsion (e.g., Bolt 1956; Lambe 1960; Skempton 1960; Sridharan and Rao 1973). It is well established that these forces play an important role in the structure of fine-grained soils such as clay and often dominate the corresponding macroscopic strength, deformation, and fluid flow behavior. Classical studies on unsaturated soils have demonstrated that capillary forces should also be considered (e.g., Bishop 1959; Lambe and Whitman 1969; Mitchell 1976). The magnitude and behavior of capillary forces are complex functions of the soil properties (e.g., particle and pore size), degree of saturation, pore air and pore-water pressure, or matric suction ($u_a - u_w$), and the properties of the multiphase fluid interface (e.g., air–water surface tension, contact angle). Manifestation of capillary forces to the macroscopic engineering behavior of unsaturated soil is readily apparent by associated increases in shear and tensile strength or by volume changes commonly observed under changing moisture conditions (e.g., collapse). The true effective stress in saturated or unsaturated soil, therefore, should more generally include macroscopic stresses such as total stress, pore air pressure, and pore-water pressure, as well as microscopic interparticle stresses such as those arising from physicochemical and capillary forces.

There are currently three widely recognized macroscale

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approaches for describing the state of stress in unsaturated soil: (1) the modified effective stress approach, which is generally attributed to the work of Bishop (1959); (2) the independent stress state variable approach, which is generally attributed to the work of Fredlund and Morgenstern (1977); and (3) modified stress variable approaches adopted by a number of researchers for stress–strain analyses.

Bishop's effective stress approach involves a modified form of Terzaghi's classic effective stress Eq. (1) written as follows:

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \tag{3}$$

where the "effective stress parameter" χ is generally considered to vary between zero and unity as a function of the degree of pore-water saturation. The difference $(\sigma - u_a)$ is the net normal stress and the difference $(u_a - u_w)$ is matric suction. For χ equal to zero (corresponding to perfectly dry conditions) and for χ equal to unity (corresponding to saturated conditions), Eq. (3) reduces to Terzaghi's effective stress for air-or water-saturated soil. For χ between zero and unity, the second term in Eq. (3), $\chi(u_a - u_w)$, describes the contribution of matric suction to effective stress. Following Bishop's approach, the macroscopic engineering behavior of unsaturated soil is described using the effective stress defined by Eq. (3) within the established framework of saturated soil mechanics. Shear strength, for example, may be described by incorporating the modified effective stress expression into the classical Mohr–Coulomb failure criterion

$$\tau_f = c' + [(\sigma - u_a) + \chi(u_a - u_w)]\tan \phi' \tag{4}$$

where c' = effective cohesion intercept; and ϕ' = effective angle of internal friction.

Following Fredlund and Morgenstern's independent stress state variable approach, net normal stress $(\sigma - u_a)$ and matric suction $(u_a - u_w)$ are treated independently in terms of their roles in the mechanical behavior of unsaturated soil. Macroscopic behavior is described in terms of the independent stress state variables and conjugate material properties. Shear strength, for example, may be described as

$$\tau_f = c' + (\sigma - u_a)\tan \phi' + (u_a - u_w)\tan \phi^b \tag{5}$$

where the first two terms comprise the classical Mohr–Coulomb criterion and the third term introduces ϕ^b as an additional friction angle to capture the contribution of matric suction to shear strength. Similar expressions have been proposed for predicting volume change.

The effectiveness, validity, and practicality of these two different approaches for describing the state of stress and corresponding behavior of unsaturated soil remain largely uncertain. Difficulties associated with experimentally or theoretically determining the effective stress parameter χ have limited the general applicability of Bishop's approach in research and practice. Experimental studies have suggested the non-uniqueness of $\chi = f(S)$. Similar experimental and conceptual difficulties associated with determining necessary material variables such as ϕ^b and uncertainties in their uniqueness over a wide range of saturation have limited the practical applicability of the independent stress variable approach. A variety of alternative approaches for stress–strain analyses have been offered in the form of modified stress variables. Matyas and Radhakrishna (1968), for example, explicitly accounted for the contribution of surface tension to intergranular stress. Alonso et al. (1990) expanded the concept of critical state soil mechanics to include volumetric strain due to matric suction. Gallipoli et al. (2003) proposed a stress variable that depends on both the degree of saturation and matric suction for elasto-plastic analysis. Houlsby (1997) illustrated that although the choice of stress state variables for unsaturated soil following phenomenological approaches could be subjective, the strain variables should be properly identified by the principle of work conjugacy. Identifying the most appropriate and practical approach for conceptualizing and quantifying the state of stress in unsaturated soil and predicting its corresponding macroscopic strength and deformation behavior remains a highly active area of research.

This paper utilizes a series of particle-scale force analyses to distinguish and conceptualize three types of forces in saturated or unsaturated soil: (1) active "skeletal" forces propagated through the soil grains; (2) active "local" forces concentrated at or near the interparticle contacts; and (3) passive particle–particle contact forces which serve to counterbalance the skeletal and local forces. For saturated soils, consideration of only the first and third types of force is often sufficient because the soil–water system may be treated as an equivalent continuum medium with macroscopic stresses defined at the boundary, as captured in Terzaghi's classic effective stress equation. Under unsaturated conditions, however, distinguishing between these three types of forces becomes necessary because pore pressure as a macroscopic stress disintegrates into several microscopic interparticle forces acting within the vicinity of the grain contacts. It is proposed here that the second type of force, which includes van der Waals forces, electrical double-layer forces, cementation forces, surface tension forces, and forces arising from negative pore water pressure, can be conceptually lumped into a macroscopic stress referred to as "suction stress." Suction stress characteristically depends on degree of saturation, water content, or matric suction through the suction stress characteristic curve (SSCC).

Physical Origin of Suction Stress

Local interparticle forces (Type II) in saturated soil include van der Waals attraction, electrical double-layer repulsion, and the net attraction arising from chemical cementation at the grain contacts. Because these forces depend jointly on both the physical and chemical properties of the soil–water system (e.g., mineralogy, surface area, pore-water chemistry), they are usually referred to as physicochemical forces. In unsaturated soil, local interparticle forces include these physicochemical forces as well as additional attractive forces arising from surface tension at air–water interfaces and attractive forces arising from typically negative pore-water pressure.

Each of the above forces may be considered "active" in nature, meaning that it does not develop in response to external forces responsible for the total stress or pore pressure, but rather arises independently from other physical and/or chemical mechanisms. For mechanical equilibrium, the local interparticle forces must be counterbalanced by "passive" particle-to-particle contact forces such as Born's repulsion and steric repulsion (Type III). As a result, local interparticle forces do not propagate from one soil grain to another through the granular skeleton, which is likely the reason that they are not explicitly considered in the macroscopic conceptualization defining Eq. (1).

Terzaghi's effective stress only accounts for forces propagating through the soil skeleton from one grain to another grain (Type I) and subtly excludes local (Type II) interparticle forces. Santamarina et al. (2001) employed a thought experiment to illustrate that the effective stress defined by Eq. (1) is not a particle-level phenomenon; rather it is established at the boundary. Pore pressure in saturated soil is, as Terzaghi suggested, a "neutral" stress, meaning that it is isotropic and invariant in direction, thus having no shear component. With the exception of the interparticle contact areas, which are usually quite small, pore-water pressure in a saturated system acts over the entire surface of the grains. Saturated soil, therefore, may be treated as an equivalent continuum medium and the stress state variables that control engineering behavior may be defined, and in many cases measured or controlled, on a macroscopic level. The macroscopic stresses that qualify for quantifying Terzaghi's effective stress, for example, are total stress and pore-water pressure. The effects of the local interparticle forces that do indeed exist have conventionally been captured in yield or failure criterion, such as the effective cohesion intercept in the Mohr–Coulomb criterion or tensile strength in the von Mises criterion or Griffith criterion. It is typically presumed that pore fluid chemistry and the strength of any cementation bonds remain fairly constant over a wide range of stress variation. As such, explicit considerations for local interparticle forces need not be included in the expression for effective stress.

Pore pressure is no longer a neutral stress in an unsaturated soil system. As soil desaturates, the pore pressure disintegrates into three forces acting through the liquid and air phases of the soil–water–air system: (1) air pressure acting on the dry or hydrated portions of the grain surfaces; (2) water pressure acting on the wetted portions of the grain surfaces in menisci formed near the interparticle contacts; and (3) surface tension acting along the air–water interfaces. The pore fluid distribution in unsaturated soil has an inherent anisotropic fabric that may be readily altered with changes in either the granular fabric or degree of saturation. The system is no longer an equivalent continuum medium and difficulties arise in our attempts to describe the state of stress from a macroscopic or boundary-level perspective.

Conceptualization of Suction Stress in Unsaturated Soil

Micromechanical analyses may be considered for a "representative elementary volume" (REV) to illustrate local interparticle forces under both saturated and unsaturated conditions. Figs. 1(a and b) show a REV and corresponding force equilibrium for a saturated fine-grained particle system. Particles are oriented to conceptualize a complex fabric associated with various possible interaction mechanisms, including face-to-face (FF), edge-to-face (EF), and edge-to-edge (EE) close and distant contact (van Olphen 1991). The free body diagram [Fig. 1(b)] shows corresponding interaction forces for the bottom-most particle in the REV through the wavy plane $A-A'$. The force F_t is a Type I force arising from gravity (self weight) or external loading that propagates through the granular skeleton. Because the system is saturated, the pore-water pressure u_w is isotropic and effectively acts over the entire surface of the soil grain. The force F_{pc} is a Type II local force arising from physicochemical mechanisms. This includes independent contributions from van der Waals attraction (F_{vdw}), electrical double-layer repulsion (F_{edl}), and chemical cementation effects (F_{ce}). Because F_{pc} depends on particle orientation and separation distance (close or distant contact), separate contributions may be assigned to various "contacts" captured in the REV. In Fig. 1(b), for example, two physicochemical forces have been assigned: F^1_{pc} $(=F^1_{\text{vdw}}-F^1_{\text{edl}}+F^1_{\text{ce}})$ is a net repulsive force owing to the predominantly FF interaction on the left-hand side of the REV; F^2_{pc} $(=F^2_{\text{vdw}}+F^2_{\text{edl}}+F^2_{\text{ce}})$ is a net attractive force owing to the predominately EF interaction on the right-hand side of the REV. The total physicochemical interaction force is considered the sum of these individual contributions ($F_{\text{pc}}=F^1_{\text{pc}}+F^2_{\text{pc}}+\ldots F^n_{\text{pc}}$).

Particle–particle contact forces such as Born's and steric repulsion provide a passive counterbalancing (Type III) force (F_C) acting at the short-range interatomic level. This force may be considered a passive energy barrier that prevents any two approaching soil grains from physically penetrating each other. Since the surface of soil grains are often regarded as nonsmooth and thus ideal frictional material, the normal contact force provides frictional resistance that inhibits the tendency for two adjacent soil particles to slide relative to one other. The magnitude of Born's repulsion and steric repulsion as a function of separation distance between two contacting solid particles are theoretically well characterized by the Leonard-Jones potential and surface hydration theories (e.g., Israelachvili 1992). The resultant contact force may also be deduced by summing all three types of forces involved in the REV. Considering the system in Fig. 1(b), force equilibrium without an external (Type I) force F_t, and isotropic pore pressure u_w leads to

$$F_{C0}=F_{\text{vdw}}+F_{\text{edl}}+F_{\text{ce}}=F_{\text{pc}} \tag{6a}$$

where the naught subscript denotes the specific condition of zero Type I force. Thus, the net interparticle contact force F_{C0} is completely counterbalanced by the physicochemical force originating from the summation of electrical double-layer repulsion, van der Waals attraction, and chemical cementation. Normalizing this force by the cross-sectional area A of the REV [Fig. 1(a)] leads to

$$\sigma_{C0}=\sigma_{\text{pc}} \tag{6b}$$

Physically, σ_{C0} is equal to the intergranular bonding stress that provides cohesion in saturated soil.

With the existence of an external Type I force F_t and isotropic pore pressure u_w, the force equilibrium becomes

$$F_t+F_{\text{pc}}-F_C-u_wA=0 \tag{7a}$$

which, if macroscopic stresses are defined ($\sigma_t=F_t/A$, $\sigma_{\text{pc}}=F_{\text{pc}}/A$, and $\sigma_c=F_c/A$), becomes

$$\sigma_C=\sigma_t-u_w+\sigma_{\text{pc}} \tag{7b}$$

or in light of Terzaghi's effective stress Eq. (1)

$$\sigma_C=\sigma'+\sigma_{\text{pc}}=\sigma'+\sigma_{C0} \tag{7c}$$

Eq. (7b), which has been obtained previously by Lambe and Whitman (1969) and Mitchell (1976), states that the net interparticle stress σ_C is the summation of Terzaghi's classical effective stress and the interparticle physicochemical stress. Since the source of shear resistance is mostly from intergranular frictional forces at the contacts, the net interparticle stress σ_C should be logically considered as the true intergranular stress qualifying for Terzaghi's descriptive definition of effective stress (e.g., Skempton 1960; Lambe and Whitman 1969).

Several fundamental changes occur upon transitioning from the saturated state to the unsaturated state. Not only do the physicochemical forces change dramatically as water is lost from the system, but they are also joined by other local force components: a distributed force due to pore air pressure u_a, a local capillary force due to surface tension F_{cap}, and a local hydrostatic force due to negative water pressure. Consequently, the macroscopic stress of pore-water pressure disintegrates into two local interparticle forces and one distributed force.

Figs. 1(c and d) show an REV for a conceptual unsaturated soil system and the corresponding free body diagram. Within the

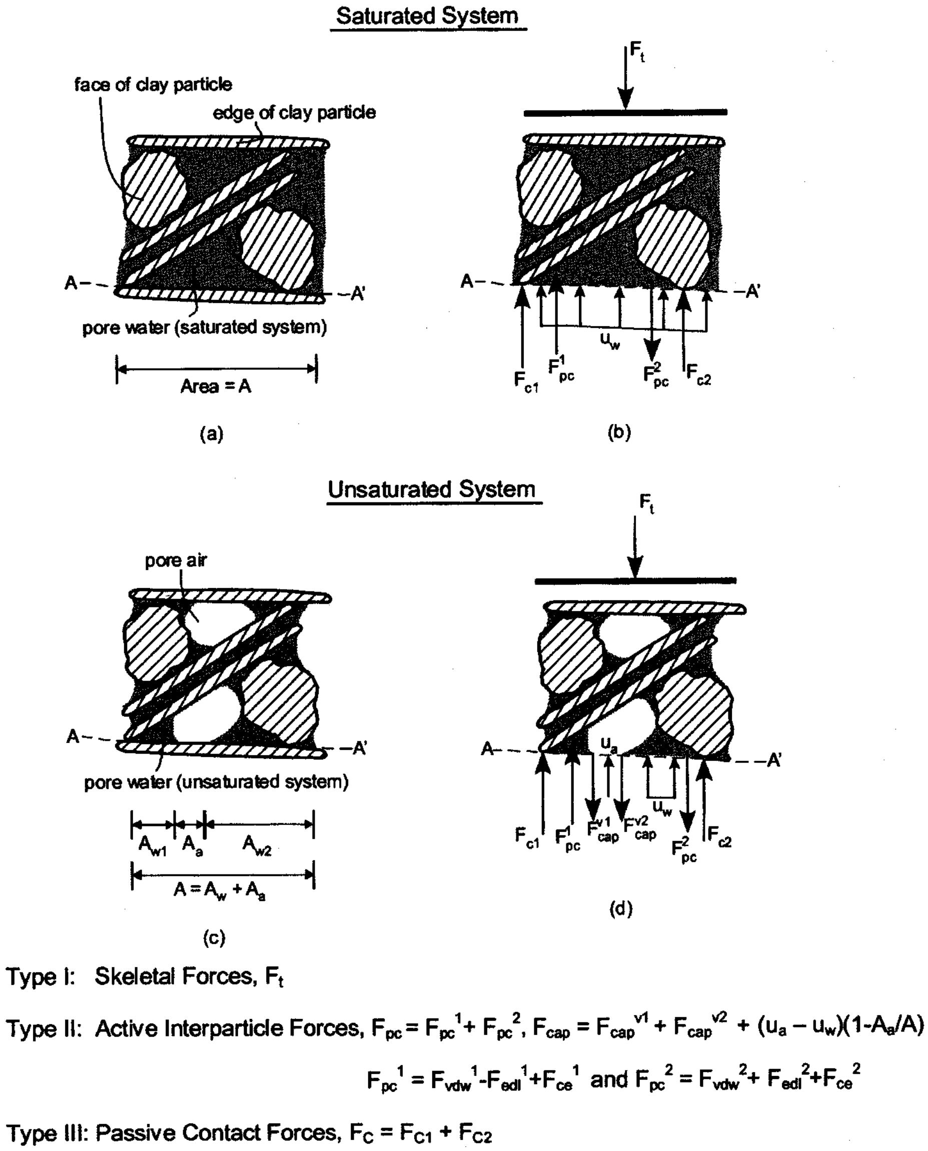

Fig. 1. Microscopic representation of interparticle force equilibrium: (a) and (b) REV for saturated fine-grained soil system, and (c) and (d) REV for unsaturated soil system. REV is chosen to represent general shape of soil particles, various particle to particle interaction modes, and force conservation. For convenience of simple force equilibrium analysis, cross-sectional area of REV is defined along wavy path A—A' to avoid cutting through particles.

funicular and pendular saturation regimes, the pore water retreats to menisci, or liquid bridges, centered about the particle contact points. Additional hygroscopic water may also be present as a thin film coating the surfaces of the solid particles under the influences of short-range hydration mechanisms. Mechanical equilibrium among all three types of forces leads to

$$\frac{F_t}{A} + \frac{F_{\rm pc}}{A} + \frac{F_{\rm cap}}{A} - \frac{F_C}{A} - u_w\frac{(A - A_a)}{A} - u_a\frac{A_a}{A} = 0 \tag{8a}$$

$$\sigma_C = \sigma_t - u_a + \sigma_{\rm pc} + \sigma_{\rm cap} + (u_a - u_w)\left(1 - \frac{A_a}{A}\right) \tag{8b}$$

where A_a=projected area where the air phase exists (A_a=$A-A_w$). The last term on the right-hand-side of Eq. (8*b*), (u_a-u_w) $\times(1-A_a/A)$, has been suggested to form the physical basis for Bishop's treatment of the contribution of matric suction to effective stress (e.g., Mitchell 1976). Therefore, we have

$$\sigma_C = \sigma_t - u_a + \sigma_{\rm pc} + \sigma_{\rm cap} + \chi(u_a - u_w) \tag{8c}$$

By introducing the term $\chi(u_a-u_w)$, Bishop indirectly captures the important role of local stresses on the stress, strain, and strength behavior of unsaturated soils. Considering the above equation, however, it becomes apparent that Bishop's effective stress neither explicitly nor effectively accounts for interparticle stresses due to physicochemical or surface tension mechanisms.

The forces arising from van der Waals attraction, electrical double-layer repulsion, and cementation may all be expected to change as desaturation commences. If the physicochemical stress at the saturated state σ_{C0} is used as a reference, a change in physicochemical stress $\Delta\sigma_{\rm pc}$ due to desaturation can be introduced into Eq. (8*c*) as follows:

$$\sigma_C = \sigma_t - u_a + \sigma_{C0} + \Delta\sigma_{\rm pc} + \sigma_{\rm cap} + \chi(u_a - u_w) \tag{8d}$$

Extending the true effective stress concept postulated by Skempton (1960) for saturated soil, the stress σ_C in the above equation may be considered the true intergranular or effective stress for unsaturated soil.

The net normal stress $(\sigma_t - u_a)$ comprising the first two terms on the right-hand side in Eq. (8*d*) is a global stress that propagates from one particle to an adjacent particle (Type I), whereas the remaining stress components are distributed local stresses that must be self-balanced between adjacent particles (Type II). The magnitudes of these components may vary along the soil grain and reach maxima at the grain contact as a part of the true intergranular stress σ_C (Type III).

It should be borne in mind that the simple one-dimensional force equilibrium analysis employed above to upscale from interparticle forces to intergranular stress is for conceptual purposes. Quantitative upscaling from interparticle forces to macroscopic stress has been an intensive research subject in the past (e.g., Bagi 1996; Oda and Iwashita 1999). The specific role of the interplay among skeletal, physicochemical forces, and pore pressure on the macroscopic stress definition is complex and depends on soil fabric. Various parallel connection models have been proposed, for example, by Lambe (1958, 1960), Balasubramonian (1972), Sridharan and Rao (1973), and Morgenstern and Balasubramonian (1980). Bolt (1956) proposed a series connection model. Hueckel (1992) reasoned that the series model by Bolt (1956) may be better suited for denser clays, whereas the parallel model by Sridharan and Rao (1973) may be better suited for loose clays. To date, a generalized upscaling model suitable for unsaturated soil remains to be found.

Magnitude of Interparticle Stress Components

Advances in interfacial physics over the past 40 years or so allow the local interparticle forces introduced above to be described in a semiquantitative way. Fig. 2(a) delineates upper bounds of interparticle stress as a function of particle size for each of the interparticle force mechanisms. Analytical equations used to obtain the bounds of these interparticle stresses are widely available in the literature (e.g., Verwey and Overbeek 1948; Ingles 1962; Rosen 1989; Israelachvili 1992; Shaw 1992). Fig. 2(b) delineates similar upper bounds for each mechanism as a function of degree of pore-water saturation.

Cementation between particles results from covalent or ionic bonds formed between the cementing agent and soil particles. The cementing agent is considered to form a new solid phase between the particles, although initially it may enter the soil system either as solid or solute prior to the cementation reaction. The magnitude of the cohesive stress occurring in soil generally varies from zero in clean sand to an upper bound of about 300 kPa (Ingles 1962). For comparison, interparticle stress due to cementation is considered constant for both cases in Figs. 2(a and b).

Van der Waals force is due to electromagnetic field interaction between adjacent atoms of approaching surfaces, and thus is cumulative. Generally, it is most pronounced at separation distances less than about 2 nm. Depending on particle size, the type and geometry of the soil solids, and the type of medium between them (e.g., pore water), van der Waals force provides a macroscopically attractive interparticle stress with an upper bound of about 1,000 kPa, as illustrated in Fig. 2(a). Since the properties of the medium between approaching soil particles (the Hamaker constant) and the associated separation distance highly depend on water content, the magnitude of van der Waals attraction, is dependent on degree of saturation and generally increases towards its upper bound at 0% saturation [Fig. 2(b)]. At large degree of saturation, the separation distance increases and van der Waals attraction decreases exponentially.

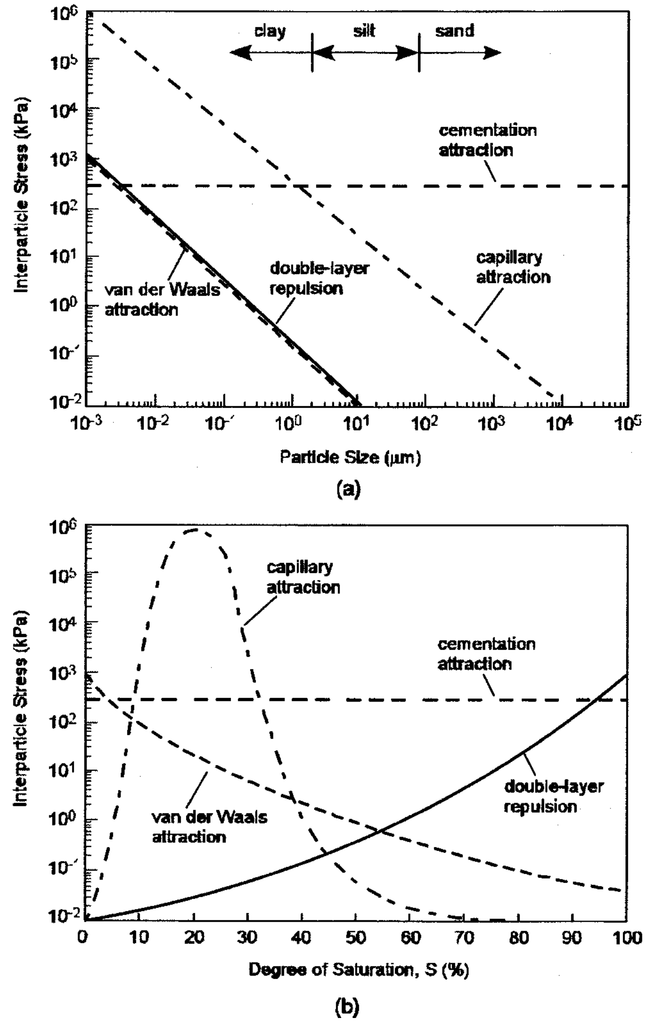

Fig. 2. Conceptual behavior and magnitude of interparticle stress components as function of: (a) particle size; and (b) degree of saturation

Electrical double-layer force is due to charge deficiency within the soil solid (clay mineral) crystalline lattice. For approaching faces of clay particles with a FF or EE arrangement, overlapping double layers generally result in repulsive interparticle stress macroscopically. Attractive double-layer interactions, although generally small, may also occur as a result of edge-to-face (EF) interparticle interaction. Depending on particle size, the type of soil solid, and the chemical constituents of the medium in the pore space, the stress due to electrical double-layer repulsion may approach about 1,000 kPa (van Olphen 1991) [Fig. 2(a)]. Double-layer forces can be significant up to separation distances of 10,000 Å (1 μm). Double-layer repulsion is most strong in clay saturated with pure water [Fig. 2(a)] and generally decreases as the pore space desaturates [Fig. 2(b)]. For coarse-grained soils such as sand, double-layer forces are essentially nonexistent.

Capillary interparticle force is marked by high variability as a function of particle size and water content, as well as the potential capacity for producing very strong attractive stresses (Santamarina et al. 2001; Lu and Likos 2004). The net force from interparticle menisci can be considered to arise from two components: one due to pore-water pressure $u_w(A - A_a)$, and one due to surface tension at the air–water interface F_{cap} [Fig. 1(c)]. Depending on the type of soil, particle size, packing geometry, degree of saturation, and the direction of wetting (i.e., along a wetting path or drying path), the magnitude of corresponding interparticle stress may approach an upper bound for extremely small particles of about 100,000 kPa (e.g., Ingles 1962; Cho and Santamarina 2001; Likos and Lu 2004) [Fig. 2(a)]. As indicated in Fig. 2(b), this stress is expected to exhibit peak behavior as a function of saturation with a maximum value occurring at some relatively low degree of saturation. The stress is zero at zero saturation, increases to some maximum as water is initially adsorbed in the form of thin films and then as capillary menisci, and then decreases toward zero as the system approaches saturation (e.g., Schubert 1975).

Contact forces such as Born's and steric repulsion are short-range forces highly dependent on interparticle separation distance and are typically significant within the range of about 5–30 Å. The contact forces not only provide mechanical balance between two adjacent particles, but are also the physical basis for interparticle compressive strength. Together with the physical and geometrical characteristics of the particle surface (e.g., mineralogy, smoothness), the contact forces provide the basis for interparticle frictional resistance and corresponding shear strength by providing a force normal to the contact area. Thus, the lower bound of separation distance between two particles determines both the normal and shear strength of the interparticle interaction. Separation distances in the lower bound range of 5 Å indicate a relatively high degree of electron shell overlap and thus higher strength, a condition likely to occur when soil is relatively dry or unhydrated.

Macroscopic Conceptualization and Determination of Suction Stress Characteristic Curve

Considering the preceding microscopic interparticle force and stress analyses [Eqs. (8*a*) and (8*b*)], an expression for the intergranular stress in unsaturated soil may be introduced as follows:

$$\sigma_C = \sigma' = \sigma_t - u_a + \sigma'_s + \sigma_{C0} \tag{9}$$

where σ'_s is herein defined as "suction stress" and is conceptualized as the resultant of interparticle physicochemical stresses attributable to cementation, van der Waals attraction, double-layer repulsion, capillary stress arising from surface tension, and negative pore-water pressure

$$\sigma'_s = \Delta\sigma_{pc} + \sigma_{cap} + \chi(u_a - u_w) \tag{10a}$$

$$\sigma'_s = \sigma_{pc} + \sigma_{cap} + \chi(u_a - u_w) - \sigma_{C0} \tag{10b}$$

As described in the preceding section, each of the interparticle stress components comprising suction stress is a function of water content, degree of saturation, or matric suction. Thus, suction stress is a characteristic function of the soil–water system described by a "suction stress characteristic curve," or SSCC

$$\sigma'_s = f(u_a - u_w) = f(S) = f(\theta) \tag{10c}$$

We may also write

$$\sigma'_s = \sigma_s - \sigma_{C0} \tag{10d}$$

where σ'_s=suction stress; σ_s="uncorrected" suction stress; and σ_{C0}=apparent tensile stress at the saturated state. Defining suction stress in this manner is intended to be consistent with Terzaghi's effective stress Eq. (1) and the so-called "apparent cohesion" concept in classical soil mechanics. For example, if the Mohr–Coulomb failure criterion is considered, the apparent tensile stress σ_{C0} can be estimated as $c'/\tan\phi'$ with c' being the apparent effective cohesion and ϕ' the effective friction angle.

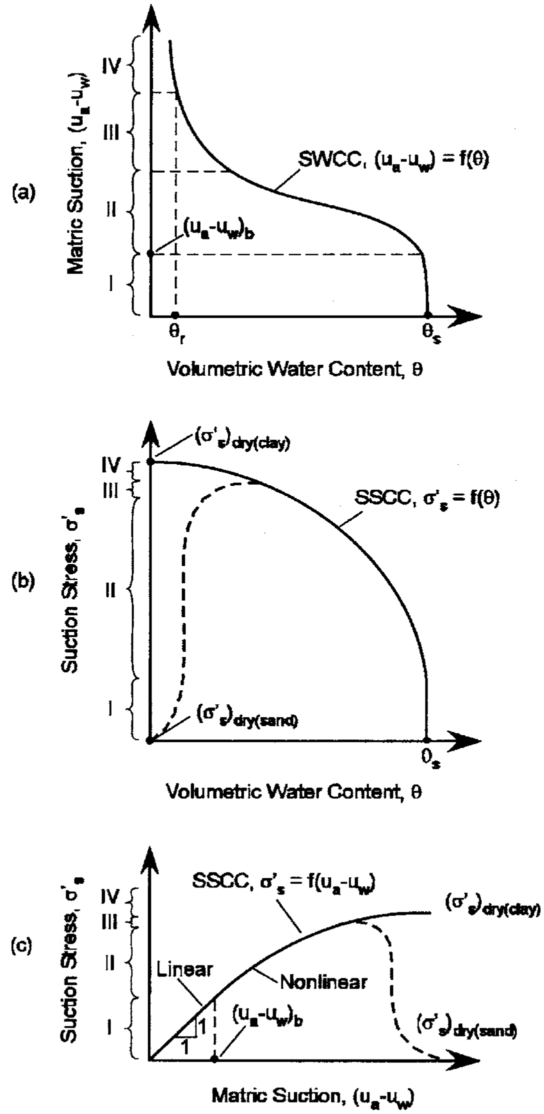

Fig. 3. Conceptual illustrations of behavioral regimes of: (a) soil–water characteristic curve, (b) suction stress characteristic curve in form $\sigma'_s(\theta)$, and (c) suction stress characteristic curve in form $\sigma'_s(u_a-u_w)$

Variation of suction and suction stress in the unsaturated state can be conceptually related to specific suction regimes as depicted in Fig. 3. Fig. 3(a) shows a typical soil–water characteristic curve ranging from the saturated volumetric water content (θ_s) at zero matric suction and the residual water content (θ_r) as the upper bound of matric suction is approached. Figs. 3(b and c) conceptualize corresponding suction stress characteristic curves in the form $\sigma'_s(\theta)$ and $\sigma'_s(u_a-u_w)$, respectively. Four regimes have been identified to differentiate the state of saturation and the dominant water adsorption mechanisms active within each. In Regime I, the system remains saturated under negative pore pressure. The upper bound of Regime I is the air-entry pressure, $(u_a-u_w)_b$. In Regime II, the "capillary" regime, pore water is retained primarily as capillary menisci located between and among the soil particles. The amount of water retained within this regime under increasing suction is primarily a function of pore size and pore size distribution. In Regime III, the amount of water retained by capillary mechanisms becomes comparable to the amount retained under short-ranged hydration mechanisms at the particle surfaces. Within Regime IV, the "residual" regime, pore water is primarily retained on the particle surfaces as water of hydration.

Within Regime I, physicochemical interparticle stress remains constant and there is no capillary interparticle stress due to surface tension because the system remains saturated and no air–water interfaces exist. Suction stress is equal to the matric suction or negative pore-water pressure [depicted by the linear 1:1 relationship in Fig. 3(c)], which operates as a neutral stress. The system, therefore, may be treated as an equivalent continuum medium.

As matric suction exceeds the air-entry value (Regime II), the soil desaturates and capillary interparticle stresses develop. Depending on soil type and pore size distribution, the water content reduces relatively rapidly with increasing suction [Fig. 3(a)], thus promoting a relatively rapid increase in capillary stress and corresponding suction stress [Figs. 3(b and c)]. Physicochemical interparticle forces change only moderately, with double-layer repulsion generally decreasing and van der Waals attraction remaining relatively unchanged since the latter is short ranged. The upper bound of matric suction for Regime II is typically within several hundred kPa for sandy soil and several thousand kPa for clayey soil. The corresponding suction stress range is typically several tens of kPa for sandy soil and several hundreds of kPa for clayey soil.

As matric suction increases beyond Regime II, the specific moisture capacity [inverse of the slope of the soil–water characteristic curve (SWCC)] becomes smaller and smaller. The amount of water retained as interparticle menisci becomes comparable to the amount retained by hydration mechanisms in the form of thin films on the particle surfaces. Regime III is marked by rapid changes in physicochemical stress as the electrical double-layer stress rapidly diminishes and van der Waals stress remains relatively unchanged. The capillary stress component also changes but is very much dependent on the type of soil. For sandy soil, capillary stress changes very little for matric suction greater than several hundred kPa, whereas for clayey soil, an additional several hundred kPa of capillary stress may be induced. The resulting suction stress varies very little for sandy soil (dashed lines), but steadily varies in clayey soil (solid lines).

As the system enters the residual regime (IV), large increases in matric suction result in only very small changes in water content [Fig. 3(a)]. Here, the majority of pore water is retained on particle surfaces as water of hydration. For sandy soil, the residual water represents monolayer coverage and typically accounts for less than a few percent by mass. For clayey soil, the residual water represents multilayer coverage and, depending on the surface area and type of clay mineral, may account for 10–25% of water content by mass (e.g., Likos 2000). The upper bound of matric suction for any soil–water–air system is approximately 10^6 kPa due to monolayer hydration bonds for silica-hydroxyls. As this upper bound is approached, capillary stress and double-layer stress drastically diminish or cease to exist, while van der Waals stress approaches a constant value. Thus, suction stress within Regime IV may diminish to zero for sandy soil but could reach several hundred kPa for clayey soil.

The magnitude of suction stress and its dependency on matric suction are evident in results from macroscopic shear strength tests. Fig. 4(a), for example, shows shear strength behavior under

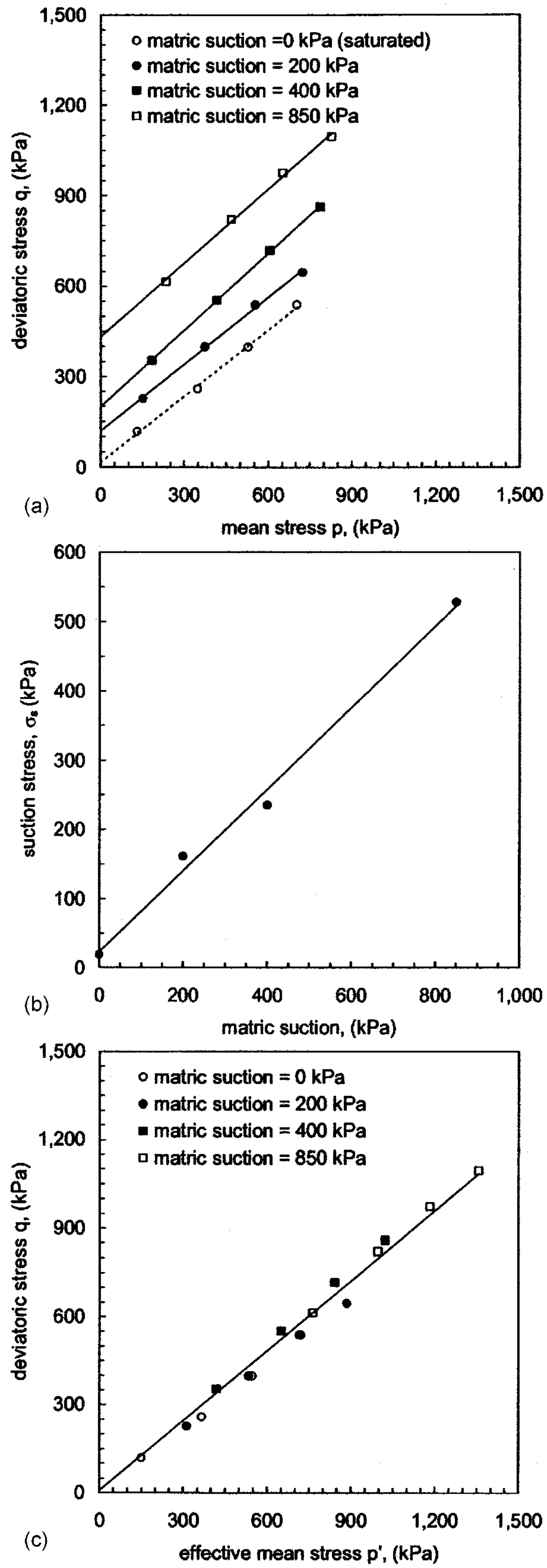

Fig. 4. Mohr–Coulomb failure for Madrid gray clay under saturated and unsaturated conditions: (a) data reduced from Escario (1980) in $p-q$ space; (b) corresponding suction stress characteristic curve, and (c) original data in $p'-q$ space [$p=(\sigma_1+\sigma_3)/2$, $p'=p+\sigma_s'$, $q=(\sigma_1-\sigma_3)$]

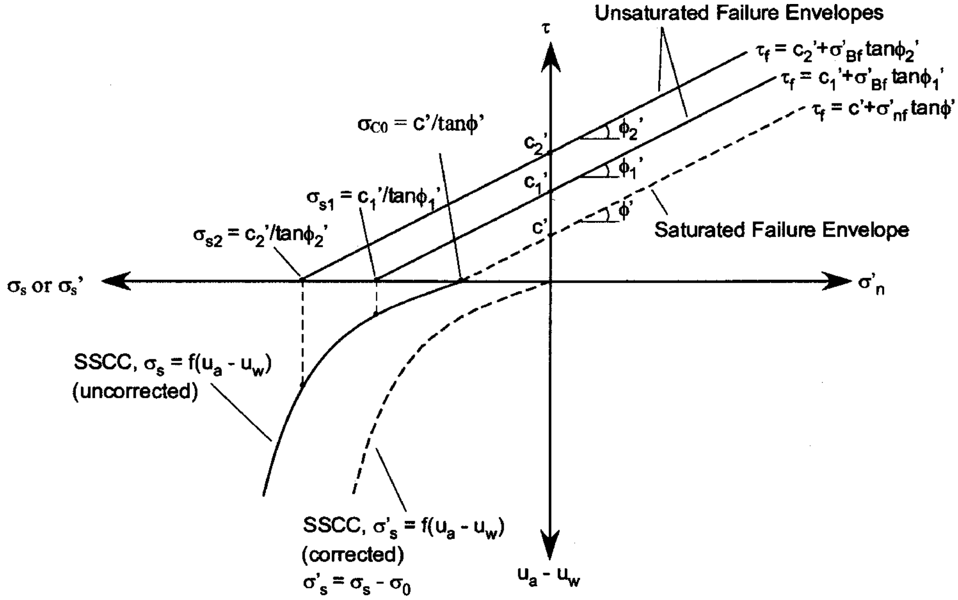

Fig. 5. Illustrated methodology for quantifying suction stress characteristic curve from Mohr–Coulomb type failure experiments

varying levels of matric suction reported by Escario (1980) from direct shear tests on Madrid gray clay. Friction angle deviates less than 4° from the saturated condition (dashed line) to the various unsaturated conditions, which is fairly typical based on synthesis of other reported experimental data (e.g., Blight 1967; Wheeler and Sivakumar 1995; Cui and Delage 1996; Vanapalli et al. 1996). Deviatoric stress at zero mean total stress (related to apparent cohesion), on the other hand, varies significantly from the saturated to the unsaturated state, increasing in Escario's results from approximately 10 kPa at saturation to about 440 kPa at a matric suction of 850 kPa. Variation in apparent cohesion has been more generally observed to depend highly on soil type and the prevailing matric suction, typically increasing in order from sand, to silt, to clay. Apparent cohesion generally varies from the saturated to unsaturated state several tens of kPa for sand and up to several hundred kPa for clay.

Projection of apparent cohesion to the normal stress axis allows apparent tensile stress to be determined, as illustrated graphically in Fig. 5. Thus, apparent cohesion evaluated from shear strength tests at specific values of matric suction may be directly used to calculate suction stress as $\sigma_s = c'/\tan\phi'$, which is a reflection of the suction stress defined by Eq. (10*c*). Suction stress as a function of matric suction (the SSCC) may then be defined by considering results from tests conducted at various levels of matric suction. The intergranular stress at specific values of matric suction may be calculated from the SSCC using Eq. (9).

Fig. 4(b) shows the SSCC calculated in this manner from Escario's original direct shear data in Fig. 4(a). If the true effective mean stress is also calculated, the failure state may be plotted in deviatoric stress-normal effective stress space, as shown in Fig. 4(c). For this soil, it is evident that the true effective stress calculated from the SSCC defines a unique Mohr–Coulomb failure envelope for each magnitude of matric suction considered, thus supporting the validity of the true effective stress and SSCC concepts.

Experimental Validation of Suction Stress Characteristic Curve

Results from unsaturated shear strength tests reported in the literature may be used to validate and demonstrate the SSCC concept. Fig. 6(a), for example, presents experimental shear strength data in the space of mean total stress and deviatoric stress for silt (Blight 1967), and glacial till (Gan et al. 1988; Vanapalli et al. 1996) at matric suction ranging from 0 (saturated) up to 350 kPa. The corresponding effective stress calculated using the suction stress concept is shown in Fig. 6(b), where the seemingly noncorrelated and scattered data points in the space of mean total stress [Fig. 6(a)] are transformed into data points falling quite closely along the Mohr–Coulomb failure lines corresponding to the saturated state for each soil. The small deviations away from the Mohr–Coulomb failure envelope observed for the shear strength data of Blight (1967) and Vanapalli et al. (1996) [Fig. 6(b)] are attributable to the slight change in friction angle upon transitioning from the saturated to unsaturated state. For the constant friction angle assumed in the shear strength data from Gan et al. (1988), effective stress based on the SSCC concept matches the Mohr–Coulomb failure envelopes with no deviation.

Fig. 7 shows another three sets of shear strength results obtained for clayey soil types (Escario 1980; Escario and Sáez 1986). Matric suction for these tests varied from 0 to 800 kPa. Again, the relatively scattered data [Fig. 7(a)] can be transformed to closely follow along the corresponding Mohr–Coulomb failure lines for the saturated state when effective stress using the SSCC concept is considered [Fig. 7(b)].

The validity of the SSCC concept for critical state failure is examined in Fig. 8 using three sets of experimental data for compacted kaolin (Wheeler and Sivakumar 1995), compacted silt (Cui and Delage 1996), and collapsible silt (Maâtouk et al. 1995). These tests employed matric suction ranging from 0 to 1,500 kPa. As shown in Fig. 8(b), unique critical state lines under both saturated and unsaturated conditions are apparent for the kaolin and

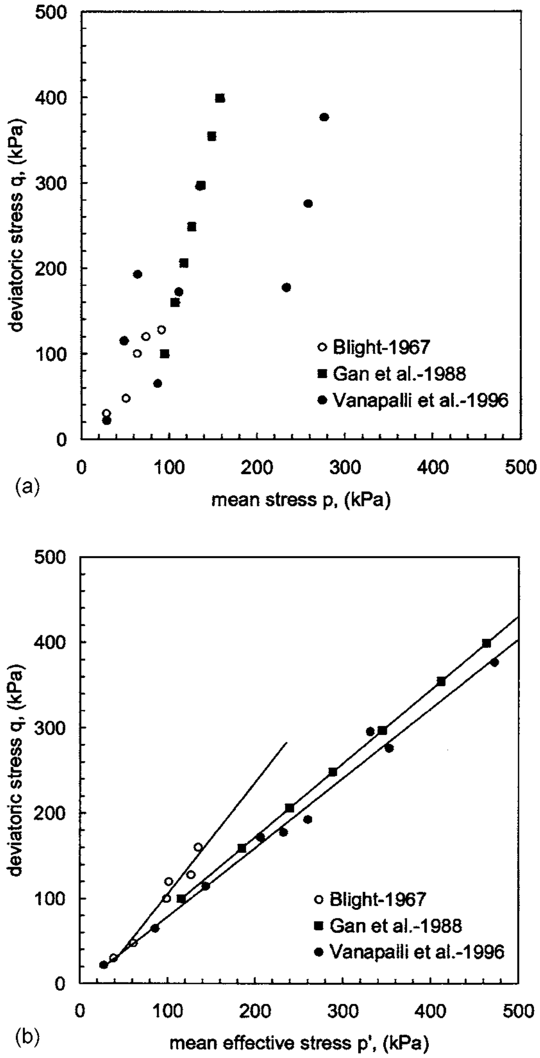

Fig. 6. Experimental validation of suction stress concept for Mohr–Coulomb failure of silt and glacial till: (a) data reduced from Blight (1967), Gan et al. (1988), and Vanapalli et al. (1996) in $p-q$ space, and (b) in $p'-q$ space

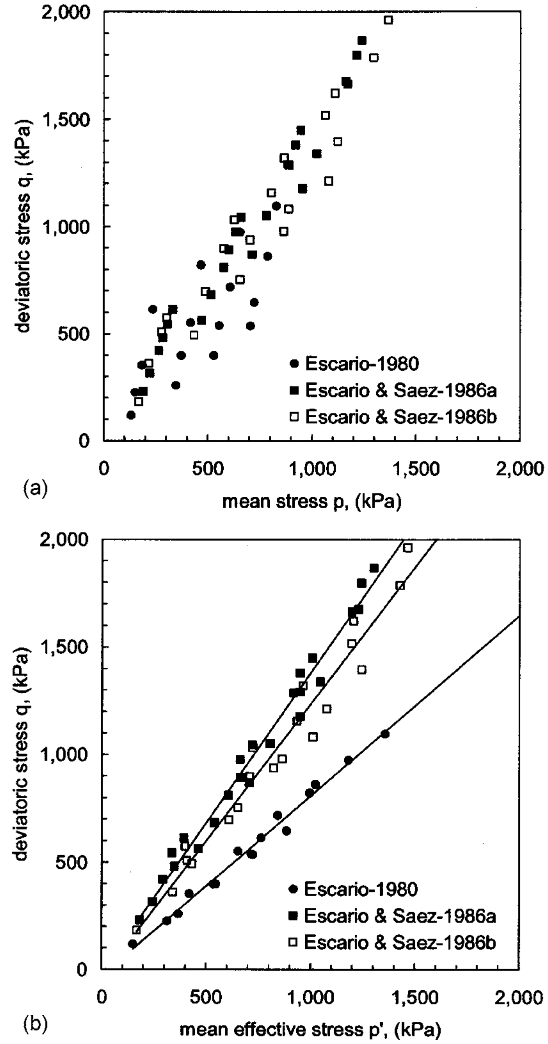

Fig. 7. Experimental validation of suction stress concept for Mohr–Coulomb failure of clayey soils: (a) data reduced from Escario (1980), and Escario and Sáez (1986) in $p-q$ space, and (b) in $p'-q$ space

collapsible silt. The critical state line is reasonably captured for each soil.

Suction stress characteristic curves $[\sigma_s = f(u_a - u_w)]$ calculated from the experimental data illustrated in Figs. 6(a), 7(a), and 8(a) are shown in Fig. 9. In general, the SSCC follows a power function varying from zero at the saturated state to no more than several hundreds of kPa. Suction stress also appears to approach a constant value at high matric suction, indicating that a practical upper bound may exist for typical unsaturated soils.

Implications of Suction Stress Characteristic Curve Concept

True effective stress calculated following the SSCC concept respects the classical effective stress definition in saturated soil mechanics, i.e., the effective stress is "that part of the total stress which produces measurable effects such as compaction or an increase of the shearing resistance" (Terzaghi 1943). Because the SSCC directly describes the state of interparticle stress, a more general approach to stress and strain problems becomes possible by eliminating the requirement to focus on a specific failure criterion or mechanism. Following the continuum mechanics formalism, deformation and stability have conventionally been analyzed as a direct result of changes in the state of stress. Following the SSCC concept, this state of stress is considered a characteristic function of unsaturated soil, thus paralleling the widely accepted concepts of the soil–water characteristic curve for describing suction-water content characteristics and the hydraulic conductivity function for describing suction-hydraulic conductivity characteristics.

Because the concept of effective stress forming the basis for classical soil mechanics is preserved within the framework of the SSCC, most conventional soil mechanics theories may be directly extended to embrace unsaturated conditions. Lu and Likos (2004), for example, presented a theory for directly applying the suction stress concept to calculate lateral earth pressure. Likos and Lu (2004) examined hysteresis in capillary-induced interparticle stress by considering stresses within a micromechanical theoretical framework based on the suction stress concept. A recent study by Heath et al. (2004) demonstrated that it is possible to expand classical soil mechanics theory and the effective stress

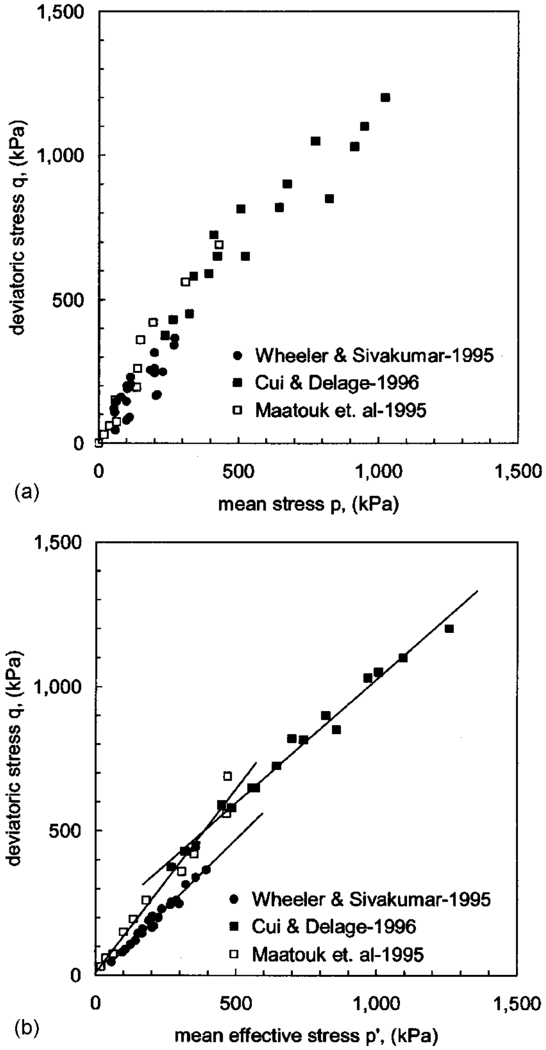

Fig. 8. Experimental validation of suction stress concept for critical state failure of clayey and silty soils: (a) data reduced from Wheeler and Sivakumar (1995), Cui and Delage (1996), and Maâtouk et al. (1995) in $p-q$ space, and (b) in $p'-q$ space

concept to describe the stress–strain behavior of unsaturated pavement subgrade materials. Lu and Griffiths (2004) used the suction stress concept to model effective stress profiles in near-surface unsaturated soil deposits under steady state infiltration and evaporation conditions.

The fact that suction stress appears to approach an upper bound at high suction bears practical and theoretical importance to existing conceptualizations for the state of stress in unsaturated soil. For clayey soils, for example, matric suction ranging from about 500 to 100,000 kPa remains important for flow behavior. It is unlikely, however, that matric suction of 100,000 kPa will result in a suction stress with the same order of magnitude. It must be recognized that matric suction cannot wholly contribute to the state of stress unless the system remains saturated and thus remains an equivalent continuum medium (i.e., for suctions less than the air-entry value). It may be argued, therefore, that use of matric suction as an independent stress state variable (Fredlund and Morgenstern 1977) may not be conceptually or physically convenient. A similar contention was offered by Khalili et al. (2004), who argued that matric suction is a pore-scale concept, whereas net normal stress is a macroscopic concept. They further

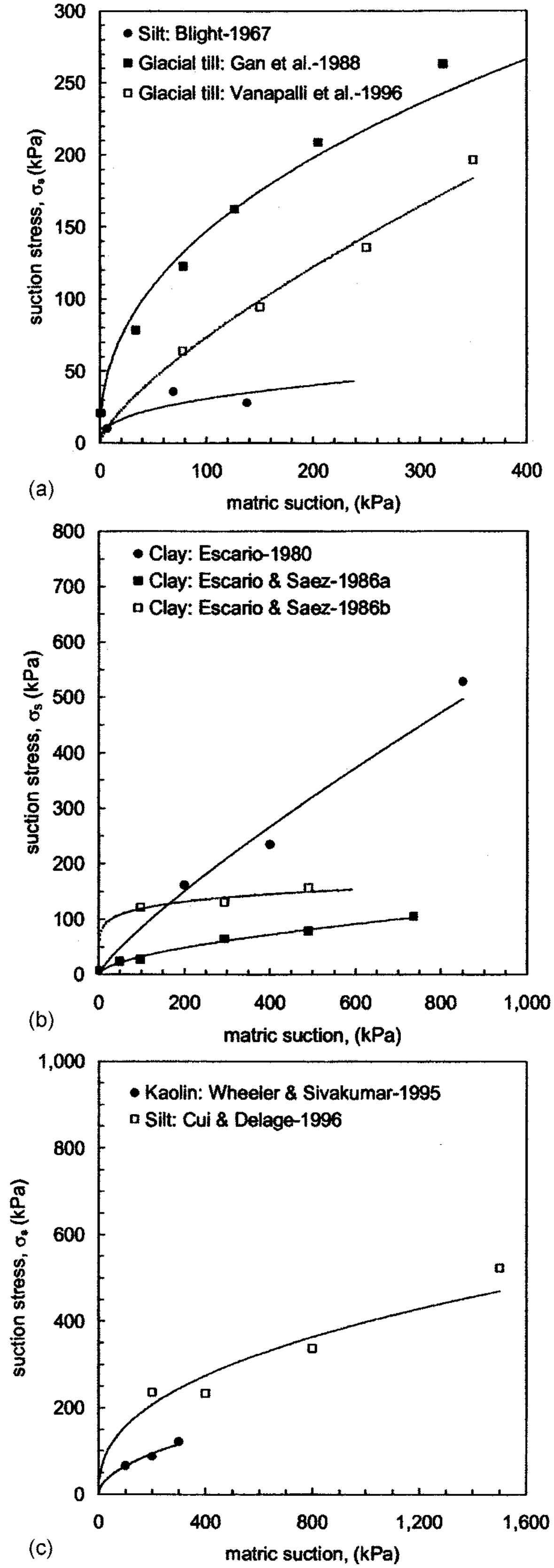

Fig. 9. Suction stress characteristic curves: (a) data reduced from Blight (1967), Gan et al. (1988), and Vanapalli et al. (1996); (b) data reduced from Escario (1980) and Escario and Sáez (1986); and (c) data reduced from Wheeler and Sivakumar (1995) and Cui and Delage (1996)

argued that using matric suction and net normal stress as independent stress state variables requires a mixing of scales and is inconsistent with the continuum mechanics formalism for multiphase systems. Bishop's (1959) effective stress conceptualization, on the other hand, requires knowledge of both the magnitude of matric suction and the effective stress parameter χ. At large suctions, suction stress could reach several hundreds of kPa, but the effective stress parameter χ intended to quantify the matric suction's direct contribution to the state of stress could be very small, making its determination subject to inherent practical uncertainty, experimental difficulty, and leading to inaccurate calculation of effective stress. The framework proposed here for assessing effective stress through the SSCC completely eliminates all of these conceptual misrepresentations and practical limitations.

The SSCC can be directly quantified through conventional unsaturated shear strength tests with no modification (e.g., matric suction control by axis translation in triaxial or direct shear systems). Because the SSCC may also be described as a function of water content or degree of saturation, conventional laboratory shear strength testing procedures may be significantly simplified by conducting water-content-controlled tests rather than suction-controlled tests. By describing the SSCC as a function of water content, the potential for real-time field monitoring of the state of stress in unsaturated soil is significantly enhanced because our current capability to measure water content in situ (e.g., using time-domain reflectometry far exceeds that of measuring suction in situ (e.g., using tensiometers or psychrometers).

Summary and Conclusions

The concept of the SSCC is proposed based on a micromechanical interparticle force consideration and the classical concept of effective stress. Three types of intergranular forces are identified: (1) active forces transmitted through the soil grains; (2) active forces at or near interparticle contacts; and (3) passive, or counterbalancing, forces at or near interparticle contacts. Suction stress, together with net normal stress, completely defines the effective stress in unsaturated soils due to all known physical stresses, including total stress, pore-air pressure, physicochemical stresses, chemical cementation, surface tension, and negative pore-water pressure. The true intergranular or effective stress for unsaturated soils is derived and defined by Eq. (9), where it consists of the net normal stress, the bonding stress when soil is saturated, and suction stress that is a characteristic function of unsaturated soil. The existence and behavior of the SSCC are validated and demonstrated by considering unsaturated shear strength testing results for a variety of soil types in the literature. Effective stress computed by considering the SSCC well represents both Mohr–Coulomb failure and critical state failure. The SSCC can be experimentally determined from conventional unsaturated shear strength tests. Since suction stress and the associated SSCC represent the actual interparticle stress in unsaturated soil, the suction stress concept is a natural extension of the classic effective stress principle. As such, most conventional soil mechanics theories may be directly extended to embrace unsaturated conditions. The SSCC is an intrinsic characteristic function of unsaturated soil, thus paralleling the well-established concepts of the SWCC for describing suction-water content characteristics and the hydraulic conductivity function (HCF) for describing suction-hydraulic conductivity characteristics. The SSCC, together with SWCC and (HCF), may fully define the characteristic functions necessary to describe transient stress phenomena in unsaturated soil.

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