An analytical assessment on the impact of covers on the onset of air convection in mine wastes

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SUMMARY

If a mine waste pile is left open, an active chemical reaction of oxidation is often found due to the commonly high content of pyritic materials. The oxidation of pyrites is an exothermic process and the released heat will promote the flow of fresh oxygen from the surrounding atmosphere into the waste dump. As a result, oxidation reaction will accelerate and temperature within the dump can increase to as high as 60°C above the ambient temperature. The oxidation process also releases sulphuric acid and hydrogen ions into ground water to cause water contamination. Low-permeability covers such as clay liners have been recently proposed to abate the oxidation process in mine wastes. The effectiveness of using low-permeability materials to cover mine wastes in order to suppress the pyrite oxidation is examined. By conducting the theoretical analysis of the onset of convective air flow within waste rocks, the conditions under which soil gas flow is significant are identified. By comparing the results with previous field measurements and theoretical analysis for the uncovered conditions, it is shown that low-permeability covers can effectively suppress soil gas flow and slow down the pyrite oxidation process in mine wastes. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: air convection; heat transfer; mine wastes; permeability; analytical solutions

INTRODUCTION

Mine wastes, if left uncovered, can release sulphuric acid to surface or ground water due to oxidation of pyrites commonly found abundant in mine wastes. Oxidation of pyrite, an exothermic process, can release as much as 375 kcal of heat for each mole of pyrite [1]. If the heat capacity of mine wastes is 240 cal/kg, this amount of heat is sufficient to raise the temperature of one kilogram of waste rocks by 1.6°C or is enough to vaporize 0.7 gram of water. Therefore, the heating within a waste rock dump undergoing active oxidation can be significant. Field measurements by Harries and Ritchie [2] and Gélinas et al. [3], showed that the temperature within mine wastes can be as much as 60°C above the ambient atmosphere temperature.

Figure 1 conceptually illustrates a typical process of acid mine drainage in a waste rock dump. The size of a waste rock dump ranges from several hundred meters to several kilometers in width.
and tens of meters in depth, making its size in tens of millions of cubic meters. Because waste rock dumps usually stand above the ground surface, all or part of the dump may be unsaturated. The typical shape of a waste dump is a flat surface with sloped sides as depicted in Figure 1. Due to the nature of mining activities, mine wastes usually are very loose and have high porosity. Therefore, oxygen in the surrounding environment can easily diffuse or flow into the dump, if it is left uncovered. Under ordinary ambient temperature conditions (0–50°C), oxidation will take place and sulphuric acid, hydrogen ions, and heat will be released. Furthermore, a positive feedback mechanism exists between heat production and air flow [4], because heating provides a driving force for air flow [5] and air flow in turn enhances the supply of oxygen [6].

Extensive work has been done in identifying and studying the controls on the acid generation process (e.g. [7,8]). The major controls include oxygen transport, chemical-kinetic and biochemical effects on oxidation rates, heat source strength, and ground water flow. The form of oxygen supply from the atmosphere has been identified in two principal mechanisms: air diffusion [9–12], and convection [7,13]. Based on quantitative study of air transport mechanisms, Bennett et al. [7] and Lu and Zhang [6] concluded that convection will not occur when the intrinsic permeability of the waste is less than $10^{-10} \text{m}^2$, which unfortunately exceeds permeability values for most waste rocks. The intrinsic permeability $k$, gravitational constant $g$, air density $\rho$, air viscosity $\mu$, and hydraulic conductivity $K$ can be related through the relationship $K = k \rho g / \mu$. To suppress the oxygen supply, several measures have been proposed recently. For example, Guo and Parizek [14] proposed to keep waste rocks under the water table so that no oxygen will be transported through air convection and diffusion. However, this technique is not feasible for many in situ or existing conditions. An alternative approach using low-permeability materials to cover waste rocks has been proposed [15–17]. It is expected that low-permeability covers will effectively suppress air convection, although oxygen transport through diffusion may still be effective.
The impact of low-permeability covers on the air convection within waste rocks is the subject of this paper. Specifically, conditions under which convective air flow can occur within waste rocks will be delineated and quantified. These conditions can be compared with those where waste rocks are left open, and the importance of low-permeability covers can then be assessed quantitatively.

Thermal convection in fluids heated from below is a classical problem and has been addressed extensively in both pure fluids and porous media. Studies [18–22] on the onset of thermal convection showed that heat transfer and fluid flow combine to form a complicated, coupled process. However, whether convective flow will occur depends on a dimensionless number, called ‘Rayleigh number.’ A system’s Rayleigh number is generally defined by parameters such as air permeability, thermal conductivity, and thickness of the porous layer. Coupled heat transfer and fluid flow in unsaturated media has been a relatively unknown area in the past. Because the governing equations for unsaturated air flow and heat transfer are highly non-linear, it is a difficult and challenging problem. Tsang and Pruess [23] studied thermal convection near a high-level nuclear waste repository in a partially saturated porous medium where rock temperature may exceed 300°C. Recently, Lu and Zhang [6] studied the onset of thermal convection in unsaturated mine wastes and concluded that air convection is a dominating mechanism for oxygen transport in most waste rocks.

In this paper, the onset of thermal convection when the low-permeability covers are applied is examined using a simple geometry model of an infinite horizontal layer of mine wastes filled with moist gas. Generally, air flow and heat transfer within mine wastes can be in three distinct regimes: conduction dominated in which oxygen can only be supplied by diffusion through covers and mine wastes, stable convection, and chaotic. In the later two regimes, oxygen can be transported by diffusion through covers and convection through mine wastes. The transitions among these regimes are the focal interest of this paper and can be found by stability analysis of the governing air flow and heat transfer equations. The stability analysis is conducted using a perturbation technique, which is the common method of solving convective instability problems (e.g. [24,25]). It involves three mathematical steps. First, the governing equations are solved assuming no convective flow to yield the so-called static or conductive solution. Second, the static solution is perturbed slightly in as general a manner as possible consistent with the boundary conditions. At this step, appropriate dimensionless quantities are identified and the perturbation equations are reformulated as an eigenvalue problem. Finally, the well-defined eigenvalue problem can be solved to describe the evolution of the perturbations with expressions which are exponential in time. The results can reveal how important the parameters such as air permeability, the thermal conductivity, and the diffusion coefficient of oxygen for mine wastes will be to the onset of air convection within mine wastes under the covered conditions.

AIR FLOW INSTABILITY ANALYSIS

**Governing equations**

Coupled air flow and heat transfer have been studied by many researchers. The governing equations employed in this study were derived by Amter et al. [26]. They consist of four fundamental equations: a constitutive relation for ideal gas, Darcy’s law, a volume balance,
and an energy balance, as follows:

\[ \rho = \frac{1}{RT} [P, \Omega_v + (P - P_v)\Omega_a] \] (1)

\[ \mathbf{q} = -\frac{k}{\mu} \nabla P g \rho z \] (2)

\[ \nabla \cdot \mathbf{q} - \mathbf{q} \left[ \left( \frac{1}{T} + \frac{1}{P_a} \frac{dP_v}{dT} \right) \nabla T - \frac{1}{P_a} \nabla P \right] = 0 \] (3)

\[ K_v \nabla^2 T - c_p^{\text{gas}} \rho q \cdot \nabla T + \frac{1}{c} \left( 1 + \frac{P_v}{P_a} \right) q \cdot \nabla P_a + S_h \]

\[ - \frac{H_v \Omega_v}{RT} q \left[ \left( 1 + \frac{P_v}{P_a} \right) \frac{dP_v}{dT} \nabla T - \frac{P_v}{P_a} \nabla P \right] = c_p^{\text{rock}} \rho_{\text{rock}} (1 - n) \frac{\partial T}{\partial t} \] (4)

where \( \rho \) is the gas density, \( R \) is the gas constant, \( T \) is the temperature, \( \Omega_v \) and \( \Omega_a \) are the molar weights of water vapor and dry air, \( g \) is the acceleration of gravity, \( k \) is the intrinsic permeability of waste rock, \( \mu \) is the viscosity of the gas, \( q \) is the air flux, and \( z \) is a downward-pointing unit vector. In the heat balance equation (4), \( K_v \) is the thermal conductivity of the porous medium, \( c \) is a conversion factor of \( 4.18 \times 10^7 \) erg cal\(^{-1}\), \( c_p^{\text{gas}} \) is the specific heat of gas at constant pressure, \( c_p^{\text{rock}} \) is the specific heat of rock, \( H_v \) is the heat of vaporization of water, \( n \) is the porosity, and \( S_h \) is an internal heat source. In this study, we assume that the relative humidity within mine wastes remains at 100 per cent [27]; thus the vapor pressure of water \( P_v \) depends only on temperature and their relationship can be found from standard steam tables. We assume that the total air pressure is the summation of dry air pressure and vapor pressure, i.e. \( P = P_a + P_v \). The heat balance equation (4) is specialized for a porous medium filled with moist gas [26]. The first term of the left-hand side represents heat conduction, the second represents sensible heat transfer, the third represents work done when the gas changes volume, and the fourth represents latent heat transfer. The right-hand side represents heating of the rocks; a term representing heating of the gas in place has been dropped because the density of gas is usually much smaller than that of rock. For a given heat source distribution \( S_h \) (see discussion below) and initial and boundary conditions, Equations (1)–(4) can be solved for field of air density \( \rho \), pressure \( P \), temperature \( T \), and gas flux \( \mathbf{q} \).

In order to quantify air flow and heat transfer in mine wastes, the heat source term \( S_h \) appearing in Equation (4) must be identified first. The mathematical form of heat production is adapted from work by Lefebvre et al. [13], which is based on analysis of field measured temperature profiles and the assumption that pyrite oxidation follows first-order kinetics with respect to oxygen concentration and that oxygen supply is by diffusion before onset. The resulting form for heat production is [13].

\[ S_h = S_o e^{-t \lambda / H} \] (5)

where \( H \) is the thickness of the waste dump, \( \lambda \) and \( \lambda \) are the heat source constants defined by

\[ S_o = FC_i K_{ox} \] (6)

\[ \lambda = \sqrt{\frac{K_{ox}}{D_{ox}}} \]
where \( F \) is the heat production constant, \( C_0 \) is the surface ambient oxygen concentration equal to \( 9.4 \times 10^{-6} \text{ mol cm}^{-3} \), \( K_{09} \) is the pyrite oxidation rate constant, and \( D_s \) is the oxygen diffusion coefficient for mine wastes. In this study gas density change due to gas composition change by chemical reaction is neglected because oxygen consumption by reaction and supply by diffusion compensate each other due to a usually slow reaction rate of oxidation in waste rocks.

**Conductive heat transfer**

Let’s start with the conductive solution of the governing Equations (1)–(4). We consider a classical problem of a porous medium bounded by one horizontal, no-heat-flow, and impermeable plane at the bottom and one horizontal, isothermal, and impermeable plane at the top as depicted in Figure 2. This problem is analogous to the Rayleigh–Bénard problem for viscous fluid and was solved by Horton and Rogers [19] and Lapwood [28] for a porous medium containing a slightly compressible fluid such as liquid water. The boundary conditions shown in Figure 2 read

\[
T = T_s, \quad P = P_s, \quad \rho = \rho_s, \quad q = 0(z = 0) \tag{7}
\]

\[
\frac{dT}{dz} = \Gamma_b, \quad q = 0(z = H)
\]

where the subscript \( s \) refers to values at the top boundary \( z = 0 \) (\( z \) points downward), subscript \( b \) values at the bottom boundary, and \( \Gamma_b \) the geothermal gradient at the bottom boundary. Solving Equations (1)–(5), and (7) yields the analytical expressions for the vertical temperature, air flux, air density, and pressure as [6]

\[
T_o = T_s + \Gamma_b z + \frac{H^2 S_o}{\lambda^2 K_r} \left[ 1 - e^{-(\lambda z/H)} - \frac{\lambda z}{H} e^{-\lambda} \right] \tag{8a}
\]

Figure 2. A simple geometry model of an infinite horizontal waste rock dump with the top surface covered with low permeability materials.
\( q_0 = 0 \)
\[ \rho_0 = \frac{\Omega_s}{RT_s} \left[ P_0 - P_{vo} \left( 1 - \frac{\Omega_s}{\Omega_v} \right) \right] \]  
\[ P_{vo} = P_{vo}(T_0) \]  
\[ P_s = P_s e^{(\theta_0/R)T_s(1/T_s)dz} \left[ 1 - \frac{g\Omega_s}{R} \left( 1 - \frac{\Omega_s}{\Omega_v} \right) \int_0^z \frac{P_{vo}}{T_0} e^{(\theta_0/R)T_s(1/T_s)dz} dz \right] \]

**Perturbation equations and eigenvalue problem**

The second step in the air flow instability analysis is to transfer the governing equations into the perturbation equations. From the field observations and the solution of other convective stability problems we expect that the conductive state will be unstable if there is a sufficiently large temperature gradient within the layer. The perturbation technique allows us to convert the governing Equations (1)–(4) into the perturbation equations for solving vertical air flux \( w \) and temperature isotherms \( \theta \) defined as follows [6]:

\[
\left( \frac{d^2}{dz^2} - \bar{D} - \bar{T}^2 \right) \theta = B_0 \bar{w} \tag{10}
\]

\[
\left( \frac{d^2}{dz^2} - (A_0 + E_0) \frac{d}{dz} - \frac{dA_0}{dz} + A_0 E_0 - \bar{T}^2 \right) \bar{w} = B_4 \bar{T}^2 F_0 \theta \tag{11}
\]

with the system Rayleigh number redefined for mine wastes as

\[
R_s = \frac{g \rho_s P_s \Omega_s e^{g_a k}}{\mu K_s RT_s} \tag{12}
\]

where

\[
A_0 = H \left( \frac{1}{T_0} + \frac{1}{P_{a0} \frac{dT_s}{dT}} \frac{dT_s}{dT} \right) \left. \frac{dP_s}{dT} \right|_{T_s} - \frac{H \frac{dP}{dT}}{P_{a0} \frac{dT}{dz}}
\]

\[
B_0 = \frac{\rho_0 H \frac{dT_0}{dz} (1 + \frac{P_{vo}}{P_{a0}}) \frac{dT_s}{dz}}{\rho \frac{dT_s}{dz}} \frac{P_s}{P_{a0}} \left[ \frac{1 + \frac{P_{vo}}{P_{a0}}} {\frac{dP_{vo}}{dT}} \frac{dT_0}{dz} - \frac{P_{vo}}{P_{a0}} \frac{dT}{dz} \right]
\]

\[
\lambda_1 = \frac{P_s}{c_p \rho_s \frac{T_s}{T_s}} \lambda_2 = \frac{H \Omega_s P_s}{c_p \rho_s \frac{T_s}{T_s}}
\]

\[ D_0 = \gamma_c \rho_{rock} R_{rock} (1 - n) \]

\[ E_0 = \frac{g \Omega_s \rho_0}{RT_0} \]

\[ F_0 = \frac{T_s^2}{P_s T_0} \left[ \frac{dP_s}{dT} \left( \frac{1}{T_s} - \frac{P_s}{T_0} \right) \left( 1 - \frac{\Omega_s}{\Omega_v} \right) + \frac{P_s}{T_0} \right] \]

\[ \bar{T} = \frac{1}{H}, \quad \bar{z} = \frac{z}{H}, \quad \bar{w} = \frac{\rho_0 H c_{p,gas} \rho_s}{K_t} w, \quad \bar{\theta} = \frac{T}{T_b - T_s}, \quad \bar{D} = \frac{H^2 D_0}{K_t} \]

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Equations (11)-(12) together with the following boundary conditions in their dimensionless form can define an eigenvalue problem for the system’s Rayleigh number \( R_e \):

\[
\begin{align*}
\theta &= 0, \bar{w} = 0; (\zeta = 0) \\
\frac{d \theta}{d \zeta} &= 0, \bar{w} = 0; (\zeta = 1)
\end{align*}
\]

(14)

The solution of Equations (10), (11), and (14) leads to an eigenvalue problem (see Appendix A for detail derivations) defined by a system of \( k \) homogeneous equations as

\[
\sum_{k=0}^{\infty} \Pi_{jk} \Theta_k = 0
\]

(15)

\[
\Pi_{jk} = (\bar{D} + f^2 \pi^2 + \bar{I}^2) \delta_{jk} + 2R_e \bar{I}^2 \int_0^1 B_0 W_k \sin \left( j - \frac{1}{2} \right) \pi \zeta \, d \zeta
\]

where \( \Theta_k \) is the coefficient of temperature solution, \( \delta_{jk} \) is the Kronecker tensor, and \( W_k \) is the solution function of the vertical flux (see Appendix A). A non-trivial solution exists when the determinant of the matrix of coefficients \( \Pi_{jk} \) vanishes, i.e.

\[
\| \Pi_{jk} \| = 0
\]

(16)

This is a \( k \)th rank eigenvalue problem. A first rank solution of the eigenvalue problem will be given by setting \( k \) equal to 1 and \( \Pi_{11} \) equal to zero and ignoring all others. This corresponds to the choice of \( \sin(\pi \zeta / 2) \) as a trial function for temperature \( \theta \). Similarly, by introducing additional terms in the expansion for \( \theta \), we will expect to get more accurate solution for the critical Rayleigh number \( R_e^* \). For the \( k \)th rank solution, Equation (15) forms \( k \)th rank determinant and can only be solved numerically for \( k > 3 \), if one attempts to obtain more accurate estimations of the system’s eigenvalues and the corresponding eigenfunctions. Because it is expected that the higher rank modes will not play an important role as the lower rank modes [18,20], calculations with second rank should give a good approximation of the stability of the system. The analytical solution for the second rank is

\[
\begin{vmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{21} & \Pi_{22}
\end{vmatrix} = \begin{vmatrix}
\bar{D} + \frac{\pi^2}{4} + \bar{I}^2 + 2R_e \bar{I}^2 I_{12} & 2R_e \bar{I}^2 I_{12} \\
2R_e \bar{I}^2 I_{21} & \bar{D} + \frac{9\pi^2}{4} + \bar{I}^2 + 2R_e \bar{I}^2 I_{22}
\end{vmatrix} = 0
\]

(17)

\[
I_{ij} = \int_0^1 B_0 W_i \sin \left( j - \frac{1}{2} \right) \pi \zeta \, d \zeta
\]

The above conditions gives the critical Rayleigh numbers corresponding the first and the second instability modes

\[
R_{e1,2}^* = \frac{\sqrt{[9\pi^2 + 4\bar{I}^2]I_{11} - (\pi^2 + 4\bar{I}^2)I_{22}}}{\sqrt{[9\pi^2 + 4\bar{I}^2]I_{11} - (\pi^2 + 4\bar{I}^2)I_{22}}^2 + 4(\pi^2 - 4\bar{I}^2)(9\pi^2 + 4\bar{I}^2)I_{11} I_{12}}
\]

(18)
Onset of convective air flow

When a waste rock dump’s Rayleigh number defined by Equation (12) is equal to the critical Rayleigh number defined by Equation (18) the waste dump is at the threshold of convection. Under the ideal condition, each waste dump has only one Rayleigh number as defined by Equation (12). Its value is linearly proportional to the system’s parameters, namely, the gravity constant $g$, the thickness of the waste dump $H$, the ambient air pressure $P_a$, the molar weight of air $\Omega$, the specific heat of gas $c_{\text{gas}}$, and the intrinsic air permeability $k$. But it is inversely proportional to the other system parameters: the viscosity of air $\mu$, the thermal conductivity of the rock $K$, the gas constant $R$, and the ambient temperature $T_s$. Therefore it is a constant for a given mine waste dump. The critical Rayleigh numbers, on the other hand, depend on the dimensionless quantities $A_0$, $B_0$, $E_0$, and $F_0$ which are not only functions of the system’s parameters, but also are strong functions of heat source constants. In general, because the strength of the heat source changes during the process of acid generation, the critical Rayleigh numbers change as time passes. If some of the critical Rayleigh numbers decrease to equal or less than the system’s Rayleigh number due to an increase in temperature gradients within waste rocks, the instability will occur and the convective air flow with patterns characterized by Figure 3 will be onset within the mine waste dump.

Figure 4 illustrates the critical Rayleigh number as a function of the horizontal wave number for different values of the maximum temperature presumingly occurring in mine wastes by using Equations (17) and (18). The critical Rayleigh number is where the critical curve reaches its minimum. For a waste dump reaching a specified maximum temperature, if the Rayleigh number is in the region above the corresponding critical curve, the system will be unstable and convective air flow will occur. Because of the non-linear dependence of vapor pressure on temperature, the higher the maximum temperature is, the lower the critical Rayleigh number will be. On the other hand, the higher the ambient temperature is, the higher the critical Rayleigh number will be.

To illustrate the onset of convective air flow, let’s construct a hypothetical scenario. When mine wastes are newly dumped, air pressure within the waste dump is in the static condition, and no oxidation has taken place. Temperature distribution within the dump is relatively uniform and temperature gradients are almost zero. Therefore, the critical Rayleigh numbers defined by Equations (17)–(18) are expected to be much larger than the system’s Rayleigh number $R_s$ defined by Equation (12). As oxygen diffuses into the rocks, pyritic materials start to oxidize and temperature gradients begin to buildup. At this early stage, conduction is still dominated within the dump and distributions of temperature and pressure can be described by Equation (8). As temperature in the dump increases, the first critical Rayleigh number defined by Equation (18) will decrease. When the temperature gradients are sufficiently large so that the first Rayleigh number is lowered to equal or smaller than the system’s Rayleigh number, the convective air flow is onset. The patterns of convective air flow and temperature distribution at this threshold moment can be characterized by the corresponding eigenfunctions and are shown in Figures 3(a) and 3(b). At this stage, only one layer of convection cells is possible within the waste dump. If the top is open to the atmosphere, convective air flow will bring more oxygen into the waste dump and will enhance the pyrite oxidation process.

As the pyrite oxidation accelerates, temperature gradients may increase to a further sufficient high level and the second critical Rayleigh number defined by Equation (18) may drop to the value of the system’s Rayleigh number. At this point the second mode may be formed and amplified and convective air flow may shift to a more active pattern illustrated in Figures 3(c).
Figure 3. Stream functions and temperature isotherms for the second rank eigenvalue problem with the top surface covered by a low-permeability layer. (a) stream functions for the first instability mode; (b) temperature isotherms for the first instability mode; (c) stream functions for the second instability mode; (d) temperature isotherms for the second instability mode. The heat source intensity constant is $S = 3.0 \times 10^{-7}$ cal cm$^{-3}$ s$^{-1}$, and the heat source decay constant is $\lambda = 3.0$. Such heat source is obtained by calibration with the field measurement data by Lefebvre et al. [13] and will create a maximum temperature of 67°C.

and 3(d). Two layers of convection cells within the dump will be observed, with the upper cells connected to the atmosphere and the bottom self-closed. Since we have an open system, the upper layer will directly promote the circulation of oxygen and air will flow in and out freely through the surface. If temperature gradients would be able to continue to increase, at some point, heat transfer will enter a chaotic regime.
Figure 4. The critical Rayleigh number as a function of the horizontal wave number and the maximum temperature with, (a) the top surface ambient temperature at 7°C, (b) the top surface ambient temperature at 27°C. Dash lines show when the top surface is open to the atmosphere (from [6]) and solid lines show when the top surface is covered by a low-permeability layer.

If, as it is being analyzed in this paper, low-permeability covers are used, several mechanisms may exist to slow down or cut off convective air flow. First, the critical Rayleigh numbers will decrease due to the boundary effect. If they drop to less than the system's Rayleigh number, air
convection will cease, and heat transfer will go back to conductive regime. Second, the low-permeability covers will dramatically decrease oxygen exchange between the waste dump and the atmosphere. And the supply of oxygen, if any, may be provided by air diffusion mechanism through the covers. The consequence of the low-permeability covers is a potential shift of oxygen supply mechanism from convection to diffusion, thereby, it may significantly suppress the oxidation process within the waste dump.

**COMPARISON WITH PREVIOUS RESULTS**

Figures 3(a) and 3(b) show, respectively, stream functions and temperature isotherms (eigenfunctions) for the first mode and Figures 3(c) and 3(d) the second mode. These streams functions and temperature isotherms represent the patterns of air flow and temperature distribution when the critical Rayleigh number reaches a waste dump’s Rayleigh number, i.e. onset of convection. Table I presents detailed results of a systematic parametric sensitivity study using parameters listed in Table II. The first critical Rayleigh number is denoted as \( R_a^* \) and the second as \( R_a'^* \). The quantity \( \sigma \), indicating the shape of convection cells, is defined as the ratio of the vertical cell length \( l_v \) to the horizontal cell length \( l_h \) as depicted in Figure 3. By comparing with the results for mine waste dumps without covers by Lu and Zhang [6] (Table I), it can be observed that for the same ambient and maximum temperatures, a covered waste dump seems to be more stable. This can be illustrated by Figure 4 (also see Table I). For example, the critical Rayleigh number for a waste rock at the ambient temperature of 27°C and the maximum temperature of 67°C is 3.85 for the open case, whereas it is 6.54 for the covered case. This implies that if the system’s Rayleigh number of a waste dump is 5.0, providing a low-permeability cover may stop air convection within the dump. It can be drawn from the parametric sensitivity study (see Table I) that the shape of the convection cells is more sensitive to the nature of the upper boundary constraints and is less affected by the maximum temperature within mine wastes and the ambient temperatures.

Table I. The critical Rayleigh numbers and \( \sigma \) (the ratio of the vertical cell length \( l_v \) to the horizontal cell length \( l_h \) depicted in Figure 3) as functions of the maximum temperature, boundary constraints, and the ambient temperatures. The heat source decay constant is specified as \( \lambda = 1.7 \) in all calculations.

<table>
<thead>
<tr>
<th>Max. temp. (°C)</th>
<th>Open at then top surface</th>
<th>Covered at the top surface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_a^* ) ( R_a' ) ( R_a^* ) ( R_a' ) ( R_a^* ) ( R_a' ) ( R_a^* ) ( R_a' )</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>21.88 293.50 43.94 678.60 37.89 331.60 81.92 779.70</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>0.53 1.34 0.61 1.28 0.76 1.51 0.79 1.45</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>11.10 144.00 15.03 222.80 18.74 161.80 27.81 254.80</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>0.53 1.36 0.52 1.31 0.75 1.53 0.77 1.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.24 77.06 7.07 99.96 10.11 86.14 12.74 113.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.54 1.39 0.52 1.32 0.74 1.55 0.75 1.49</td>
<td></td>
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<tr>
<td></td>
<td>3.76 42.93 3.85 50.25 5.74 47.72 6.54 56.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.57 1.41 0.61 1.35 0.73 1.58 0.74 1.52</td>
<td></td>
</tr>
</tbody>
</table>
Several field cases were well documented previously (e.g. [1–3]). Of them, the case reported by Gélinas et al. [3] was used by Lu and Zhang [6] for a direct comparison where the top of the mine wastes is open to the atmosphere. Field scale measurements of air pressure, temperature, and chemical concentration were conducted, and strong convective air flow was found about five years after the completion of the dump at the South Dump of La Mine Doyon, Quebec. The waste dump has similar parameters to those listed in Table II. The areal dimensions of the waste dump are 100 m × 600 m with 30–50 m in depth. Using the parameters listed in Table II the dump’s Rayleigh number $R_a$ is estimated by Equation (12) as 8.4 for the ambient temperature of 280 K (27°C). The critical Rayleigh number, as calculated in the proceeded discussion, is $R_a^* = 3.85$ for the first mode and $R_a^* = 50.25$ for the second mode for a temperature difference of 40°C (see Table I). According to the theory, convective air flow of the first mode will occur as shown in Figures 3(a) and 3(b) if the top is left open. This has been confirmed by Gélinas et al. [3]. Furthermore, the convective air flow will still occur even if a low permeability cover is provided, because the first critical Rayleigh number ($R_a^* = 5.74$) is less than the system’s Rayleigh number ($R_a = 8.4$). However, if a treatment is used to decrease the air permeability of the mine wastes by 40% such as dynamic compaction techniques, the system’s Rayleigh number can be reduced to 5.0. Based on the theory, this will cease the air convection within mine wastes because the critical Rayleigh number would becomes greater than the system’s Rayleigh number.

Another way to lower the system’s Rayleigh number in order to improve the stability of mine wastes is to reduce the thickness of waste dumps. In the case like the South Dump of La Mine Doyon, if the depth were 20 m instead of 35 m, the system’s Rayleigh number would decrease from $R_a = 8.4–5.0$ so that it is below the first critical Rayleigh number $R_a^* = 5.74$ for the covered condition. This could ensure that the first instability would not occur, given the other system parameters unchanged.

If the cover is applied before the waste dump reaches its maximum temperature 330 K (67°C), air convection may be ceased even without any reduction in air permeability. For example, if a low permeability cover is employed when the temperature of the waste dump reaches 320 K (57°C), from Table I we know that the first critical Rayleigh number is 6.24 for the open case, and 10.11 for the covered case. Therefore, if the dump is left open, convective air flow will occur because the system’s Rayleigh number (8.4) is greater than the first critical Rayleigh number (6.24). However, convective air flow will cease once a cover is applied, because the system’s Rayleigh number is less than the first critical Rayleigh number (10.11). This implies that earlier emplacements of covers are preferable and may be necessary before the mine wastes become hot.

Table II. Parameters used in the air flow instability analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{\text{gas}}$</td>
<td>$2.4 \times 10^{-1}$ cal g$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$c_{\text{rock}}$</td>
<td>$2.5 \times 10^{-1}$ cal g$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$9.8 \times 10^2$ cm$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$8.88 \times 10^5$ g cm$^{-3}$ s$^{-2}$</td>
</tr>
<tr>
<td>$H$</td>
<td>$3.0 \times 10^5$ cm$^3$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$5.39 \times 10^2$ cal g$^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>$10^{-5}$ cm$^2$</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>$40–70^\circ$C</td>
</tr>
<tr>
<td>$S_0^*$</td>
<td>$10^{-3}–40^{-8}$ cal cm$^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.0–3.0 dimensionless</td>
</tr>
</tbody>
</table>

$S_0^*$ is calculated in this study by using known values of the heat decay constant $\lambda$ and the maximum temperature within a dump.
CONCLUSIONS

The effectiveness and consequence of using low-permeability covers in mine wastes are assessed quantitatively. A stability analysis of the onset of air convection in mine wastes is conducted for the case when waste dumps are covered with low permeability materials. The governing equations for coupled air flow and heat transfer, together with the appropriate boundary conditions, are reformulated into an eigenvalue problem and are solved analytically. Conditions that distinguish different thermal regimes are thereby delineated and identified quantitatively.

The system’s Rayleigh number is the key indicator in analysis of stability problems. This parameter must be redefined for the waste rock problem because the existence of a distributed heat source within the waste dump. If the eigenvalues of the system or the critical Rayleigh numbers (defined by Equation (18)) are above the system Rayleigh number defined by Equation (12), no convection is present within the mine wastes. Under such conditions, heat conduction will be the dominating mechanism for energy transfer, and air diffusion will be the dominating mechanism for oxygen transport. However, if the eigenvalues are less than the Rayleigh numbers, convective air flow will occur beneath the covers. The critical Rayleigh numbers for a waste dump filled with moist air are more complicated than that in the classic cases of a liquid-saturated system where the critical Rayleigh number is $4\pi^2$ [19,28]. In general, the critical Rayleigh numbers of waste dumps filled with air are one order of magnitude smaller than those filled with liquid water. This is because air is much more compressible than water. From the parametric sensitivity study, it is observed that the critical Rayleigh numbers of covered cases are smaller than those of uncovered cases, which implies that providing low-permeability covers generally will stabilize mine waste dumps. Our analysis shows that convective air flow may still occur even under covered conditions, depending on a waste dump’s parameters and the atmospheric conditions. In comparison with the previous field measurements and theoretical analysis for the case where mine wastes are left open, it is found that using low-permeability covers can improve the stability conditions, therefore suppress the occurrence of convective air flow and slow down the pyrite oxidation process.

For a low-permeability cover to be effective, it is required that a waste dump’s Rayleigh number should be smaller than the first critical Rayleigh number. Such condition provides some useful guidelines for mine waste emplacement. To prevent air convection, strategies can be devised by either decreasing the waste dump’s Rayleigh number or increasing the first critical Rayleigh number. For example, a thinner layer of mine wastes and a lower waste air permeability will result in lower values of the dump’s Rayleigh number. Keeping mine waste moist may also be a feasible technique because it will increase the thermal conductivity, thereby decrease the system’s Rayleigh number. One the other hand, an earlier emplacement of covers (before a high rate of oxidation and a high temperature buildup occurs) may increase the critical Rayleigh number to above the system’s Rayleigh number to prevent the onset of convective air flow.

In this analysis, we assume that the covers are impermeable but still diffusible. The impact of the cover’s diffusivity is not examined here, but could be significant in the long-term performance of mine wastes. If low permeability covers can provide effective means to stop oxygen supply through air convection between the surrounding atmosphere and the mine wastes, the magnitude of the convective air flow and the acid generation rate within the mine wastes, if any, will depend directly on the steady-state supply of oxygen through diffusion.
APPENDIX A. ANALYTICAL SOLUTION OF THE EIGENVALUE PROBLEM

Because the boundary conditions (14) require that there is a fixed temperature on the top and no heat flow on the bottom surface, we can assume a general solution form for the temperature field as

\[ \theta = \sum_j \Theta_j \sin \left( j - \frac{1}{2} \right) \pi \zeta \]  

(A1)

Equations (10) and (11) can then be rewritten as

\[ \sum_j \left( \tilde{D} + \left( j - \frac{1}{2} \right)^2 \pi^2 + \tilde{I}^2 \right) \Theta_j \sin \left( j - \frac{1}{2} \right) \pi \zeta = -B_0 \tilde{w} \]

(A2)

\[ \left( \frac{d^2}{d\zeta^2} - (A_0 + E_0) \frac{d}{d\zeta} - \frac{dA_0}{d\zeta} + A_0E_0 - \tilde{I}^2 \right) \tilde{w} = R_a \tilde{I}^2 F_0 \sum_j \Theta_j \sin \left( j - \frac{1}{2} \right) \pi \zeta \]

Now if we insert the following form for air velocity:

\[ \tilde{w} = R_a \tilde{I}^2 \sum_k \Theta_k W_k(\zeta) \]  

(A3)

into the second equation of (A2), we have

\[ \left( \frac{d^2}{d\zeta^2} - (A_0 + E_0) \frac{d}{d\zeta} - \frac{dA_0}{d\zeta} + A_0E_0 - \tilde{I}^2 \right) W_k = F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta \]  

(A4)

which is required to satisfy the following boundary conditions according to Equation (14):

\[ W_k|_{\zeta = 0} = 0; \quad W_k|_{\zeta = 1} = 0 \]  

(A5)

Equation (A4) is a second-order differential equation with variable coefficients. To solve this equation, we first make the following transformation:

\[ W_k = e^{1/2 \int (A_0 + E_0) d\zeta} \frac{dV_k}{d\zeta} \]  

(A6)

we then have

\[ \frac{dW_k}{d\zeta} = \frac{1}{2} (A_0 + E_0) W_k + e^{1/2 \int (A_0 + E_0) d\zeta} \frac{dV_k}{d\zeta} \]

(A7)

\[ \frac{d^2W_k}{d\zeta^2} = e^{1/2 \int (A_0 + E_0) d\zeta} \left[ \frac{1}{4} (A_0 + E_0)^2 V_k + \frac{V_k}{2} \frac{d}{d\zeta} (A_0 + E_0) + (A_0 + E_0) \frac{dV_k}{d\zeta} + \frac{d^2V_k}{d\zeta^2} \right] \]

Substituting Equation (A7) into Equation (A4) yields

\[ \frac{d^2V_k}{d\zeta^2} - \frac{V_k}{2} \frac{d}{d\zeta} (A_0 - E_0) - \frac{1}{4} (A_0 + E_0)^2 V_k - \tilde{I}^2 V_k = \left( F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta \right) e^{-1/2 \int (A_0 + E_0) d\zeta} \]  

(A8)

with boundary conditions (A5) become

\[ V_k|_{\zeta = 0} = 0; \quad V_k|_{\zeta = 1} = 0 \]  

(A9)
Equation (A8) can be solved by the WKB method [29], which gives

\[ V_k = \frac{1}{\sqrt{\pi}} \left( c_1 e^{\sqrt{(f) d\zeta}} + c_2 e^{-\sqrt{(f) d\zeta}} \right) \]

\[ f = \frac{d}{2 d\zeta} (A_0 - E_0) + \frac{1}{4} (A_0 - E_0)^2 + \bar{l}^2 \]

\[ c_1 = c_1^0 + \frac{1}{2} \int_0^\zeta f^{-1/4} \exp \left( - \int_0^\zeta \sqrt{(f) d\zeta} - \frac{1}{2} \int_0^\zeta A_0 d\zeta \right) F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta d\zeta \]

\[ c_2 = c_2^0 - \frac{1}{2} \int_0^\zeta f^{-1/4} \exp \left( \int_0^\zeta \sqrt{(f) d\zeta} - \frac{1}{2} \int_0^\zeta A_0 d\zeta \right) F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta d\zeta \quad (A10) \]

The integral constants \( c_1^0 \) and \( c_2^0 \) then can be determined by the boundary conditions (A9) as

\[ c_1^0 = -c_2^0 = \frac{1}{4} \sinh^{-1} \left( \int_0^1 \sqrt{(f) d\zeta} \right) \left\{ \left[ \int_0^1 f^{-1/4} \exp \left( \int_0^\zeta \sqrt{(f) d\zeta} \right) \right. \right.

\[ - \frac{1}{2} \int_0^\zeta (A_0 + E_0) d\zeta \right] F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta d\zeta \left. \right] \]

\[ - \left. \left[ \int_0^1 f^{1/4} \exp \left( - \int_0^\zeta \sqrt{(f) d\zeta} \right) \right. \right.

\[ - \frac{1}{2} \int_0^\zeta (A_0 + E_0) d\zeta \right] F_0 \sin \left( k - \frac{1}{2} \right) \pi \zeta d\zeta \]  \left. \right\} \quad (A11) \]

To solve \( \Theta_k \), we substitute (A3) into the first equation of (A2), which yields

\[ \sum_j (\bar{D} + j^2 \pi^2 + \bar{l}^2) \Theta_j \sin \left( j - \frac{1}{2} \right) \pi \zeta = -B_0 R_0 \bar{l}^2 \sum_k \Theta_k V_k \quad (A12) \]

For the theory of the Fourier sine series transform, we have

\[ B_0 W_k = \sum_j \left( 2 \int_0^1 B_0 W_k \sin \left( j - \frac{1}{2} \right) \pi \zeta d\zeta \right) \sin \left( j - \frac{1}{2} \right) \pi \zeta \quad (A13) \]

Inserting the above equation into Equation (A12) gives

\[ \sum_j (\bar{D} + j^2 \pi^2 + \bar{l}^2) \Theta_j \sin \left( j - \frac{1}{2} \right) \pi \zeta \]

\[ + 2R_0 \bar{l}^2 \sum_j \Theta_j \left( \int_0^1 B_0 W_k \sin \left( j - \frac{1}{2} \right) \pi \zeta d\zeta \right) \sin \left( j - \frac{1}{2} \right) \pi \zeta = 0 \quad (A14) \]

For a non-trivial solution to exist, each term must vanish individually. So, the above equation becomes Equation (15) as

\[ \sum_{k=0}^\infty \Pi_{jk} \Theta_k = 0 \]

\[ \Pi_{jk} = (\bar{D} + j^2 \pi^2 + \bar{l}^2) \delta_{jk} + 2R_0 \bar{l}^2 \int_0^1 B_0 W_k \sin \left( j - \frac{1}{2} \right) \pi \zeta d\zeta \quad (A15) \]
APPENDIX B: NOTATION

\( A_0 \) dimensionless coefficient
\( B_0 \) dimensionless coefficient
\( c \) conversion factor, \( 4.18 \times 10^7 \) [erg cal\(^{-1}\)]
\( c_1^0 \) integral coefficient
\( c_2^0 \) integral coefficient
\( c_{p,\text{gas}} \) specific heat of air [cal g\(^{-1}\) K\(^{-1}\)]
\( c_{p,\text{rock}} \) specific heat of rock [cal g\(^{-1}\) K\(^{-1}\)]
\( C_o \) ambient oxygen concentration, \( 9.4 \times 10^{-6} \) [mol cm\(^{-3}\)]
\( D_v \) oxygen diffusion coefficient [cm\(^2\) s\(^{-1}\)]
\( D_0 \) time constant parameter
\( E_0 \) dimensionless coefficient
\( f \) function defined in Equation (A10)
\( F \) heat production constant [cal mol\(^{-1}\)]
\( F_0 \) dimensionless coefficient
\( g \) gravitational acceleration, 980 [cm s\(^{-2}\)]
\( H \) thickness of waste dump [cm]
\( H_v \) heat of vaporization of water [cal g\(^{-1}\)]
\( k \) intrinsic permeability of mine wastes [cm\(^2\)]
\( K_{ox} \) oxidation rate [s\(^{-1}\)]
\( K_t \) thermal conductivity of mine wastes [cal K\(^{-1}\) cm\(^{-1}\) s\(^{-1}\)]
\( l \) horizontal wave number
\( n \) porosity [dimensionless]
\( P \) air pressure [g cm\(^{-1}\) s\(^{-2}\)]
\( P_o \) defined by \( P_o = P - P_v \) [g cm\(^{-1}\) s\(^{-2}\)]
\( P_v \) vapor pressure of water [g cm\(^{-1}\) s\(^{-2}\)]
\( q \) gas flux [cm\(^3\) s\(^{-1}\)]
\( R \) gas constant [g cm\(^2\) s\(^{-2}\) mol\(^{-1}\) K\(^{-1}\)]
\( R_0 \) Rayleigh number [dimensionless]
\( S_0 \) heat source intensity constant [cal cm\(^{-3}\) s\(^{-1}\)]
\( S_h \) internal heat source [cal cm\(^{-3}\) s\(^{-1}\)]
\( t \) time [s]
\( T \) absolute temperature [K]
\( \Delta T \) temperature difference between top and bottom boundaries [K]
\( w \) dimensionless vertical air flux
\( W \) solution function of vertical flux
\( z \) vertical coordinate [cm]
\( z \) downward pointing unit vector

\textbf{Greek symbols}

\( \gamma \) time rate constant [s\(^{-1}\)]
\( \Gamma_b \) temperature gradient at \( z = H \)
\( \delta \) Kronecker delta
\( \zeta \) dimensionless vertical coordinate

\( \Theta \) coefficient of temperature solution
\( \theta \) dimensionless temperature
\( \lambda \) heat source decay constant
\( \lambda_1 \) defined by equation (13)
\( \lambda_2 \) defined by equation (13)
\( \mu \) air viscosity [g cm\(^{-1}\) s\(^{-1}\)]
\( \Pi \) defined by equation (15)
\( \rho \) air density [g cm\(^{-3}\)]
\( \rho_{\text{rock}} \) rock density [g cm\(^{-3}\)]
\( \phi \) stream function
\( \Omega_{\text{dry}} \) molar weight of dry air [g mol\(^{-1}\)]
\( \Omega_{\text{w}} \) molar weight of water [g mol\(^{-1}\)]

Subscripts

0 static solution
b quantity at bottom where \( z = H \)
s quantity at top where \( z = 0 \)
\( j, k \) rank or summation indices
1, 2 instability mode indices

REFERENCES


