Correlation between Soil-Shrinkage Curve and Water-Retention Characteristics

Ning Lu, F.ASCE; and Yi Dong, A.M.ASCE

Abstract: A soil-shrinkage curve (SSC) can be defined as the characteristic volume–water content relation under free external stress conditions. A SSC is traditionally divided into three characteristic states: normal shrinkage when soil is saturated, residual shrinkage when soil is unsaturated and subjected to volume reduction, and zero shrinkage when soil approaches the dry state. Using the measured SSC, soil water-retention (SWR) curve, and suction stress characteristic curve of various silty and clayey soils, the traditional states are found to be not well defined and incorrect, particularly when soil approaches the dry state. A conceptual model for SSC is proposed in which SSC can be divided into four states: capillary, pendular, adsorbed, and tightly adsorbed; each is governed by one or two SWR mechanisms: capillary and adsorption. The shrinkage rate, defined here as the change in void ratio due to the change in moisture ratio, is found to be not universally zero in adsorptive states. It is shown that the shrinkage rate in the adsorption SWR regime is highly correlated to SWR characteristics—namely, the specific surface area—or to cation-exchange capacity, or that total amount of adsorption water follows the same exponential form with the shrinkage rate. DOI: 10.1061/(ASCE)GT.1943-5606.0001741. © 2017 American Society of Civil Engineers.

Author keywords: Soil shrinkage; Soil water-retention curve; Unsaturated soil; Specific surface area; Cation-exchange capacity; Matric suction.

Introduction

The soil volume–water content relationship under a free-confining condition of drying is a constitutive or characteristic curve (e.g., Hamberg 1985; Perko et al. 2000). It is often called the soil-shrinkage curve (SSC) or free shrinkage curve in the literature, which has been used for understanding and quantifying soil–water interaction and volumetric behavior, particularly for expansive soil classifications (e.g., McKeen 1992; Nelson and Miller 1992; Likos et al. 2003). In recent years, the shrinkage behavior of soil during drying has been shown to influence mechanical and hydraulic soil properties (e.g., Chertkov 2003; Braudeau and Mohtar 2004). Soil shrinkage behavior can be used to determine the formation and development of cracks under various boundary conditions (e.g., Beven and Germann 1982; Chertkov 2000). Some work also demonstrated that SSC has a close relationship with the soil water-retention curve (SWRC) (Fredlund et al. 2002; Peng and Horn 2005; Lin and Cerato 2013).

The SSC can be described in several similar ways as soil volume changes with water content, including as the specific volume of soil versus its gravimetric water content (e.g., Mitchell 1992; Crescimanno and Provenzano 1999; Chertkov 2000; Boivin et al. 2006) and as the void ratio versus the moisture ratio (e.g., Kim et al. 1992; Groenevelt and Grant 2002; Peng and Horn 2005; Cornelis et al. 2006). Moisture ratio \( m \) is defined as the ratio of volume of water \( V_w \) to volume of solid \( V_s \). Because both void ratio and moisture ratio are normalized by the same quantity—the volume of solids—the latest definition of SSC, i.e., the comparison of void ratio and moisture ratio, is indicative of the volume change of soil with respect to the volume of water, and is adopted in this paper.

Traditionally, SSC has been obtained by the so-called Clod test in which soil clods from the field or laboratory are used to determine the SSC during a drying process (e.g., Hamberg 1985; McKeen 1992; Krosley et al. 2003). Because the Clod test involves manual measurement of soil volume and water content at discrete points of soil water content under natural restrained drying conditions due to the resin coating, it is time-consuming and inaccurate, particularly when the matric suction (or relative humidity) of the clod approaches the ambient suction condition of water vapor (or relative humidity). Thus little data for SSC with gravimetric water content between 5% and oven dry are available, and an assumption has been widely made that the shrinkage rate (defined as the change of void ratio divided by the change of moisture ratio), or the slope of the SSC, becomes zero as soil approaches the dry state (e.g., Bruand and Prost 1987; Bronswijk 1991; Peng and Horn 2005). Recently, new techniques have been developed for studying soil shrinkage (e.g., Abou Najm et al. 2009; Lu and Kaya 2013; Dong and Lu 2017) and soil cracking (e.g., Péron et al. 2007; Abou Najm et al. 2009; Sanchez et al. 2013). These methods generally employ high-resolution photography or laser techniques for continuous volume-change monitoring or suction control techniques for continuous control of ambient environments.

Recent advances in the understanding of the soil water-retention (SWR) behavior reveal that the fundamental soil–water interactions at different SWR regimes control the macroscopic soil properties, e.g., thermal conductivity (Lu and Dong 2015), effective stress or suction stress (Dong and Lu 2016), and small-strain shear modulus (Dong et al. 2016). This paper finds the soil shrinkage behavior to be directly correlated with SWR characteristics. A new conceptual model for SWR-based interpretation of SSC is provided and used to...
Shrinkage (Kim et al. 1992; Chertkov 2000; Peng and Horn 2005). Shrinkage (Braudeau et al. 1999; Peng and Horn 2005); and (3) zero (Groenevelt and Grant 2001); (2) residual shrinkage or transition to the completely dry state: (1) normal shrinkage (Cornelis et al. 2006; Peng and Horn 2005; Boivin et al. 2006), and geotechnical engineering (e.g., McKeen 1992; Perko et al. 2000). A typical SSC, portraying the change of void ratio in terms of moisture ratio, is illustrated in Fig. 1. Because both void ratio and moisture ratio are defined by normalizing the volume of void or liquid water by the common quantity of volume of solid, the 1:1 ratio line of void ratio and moisture ratio indicates that soil is fully saturated, and the volume reduction of soil is only due to the volume change of its liquid water.

In studying soil shrinkage behavior, soils can be classified in two categories, structured and nonstructured, based on their formation environment and hydrologic behavior. For structured soils, which contain large interaggregate pores and channels caused by natural factors such as worms or roots, the pore water would be drained first through a well-known stage called the structural shrinkage phase without considerably reducing the total volume of the bulk soil (e.g., Cornelis et al. 2006; Peng and Horn 2013). This is probably due to the fact that interparticle stress or suction stress during this stage is mostly under overly consolidated conditions. Structured soil is most relevant to agricultural and geological problems, and is not discussed further in this paper.

Newly remolded or compacted soils are considered nonstructured soils. The traditional interpretation of volume shrinkage behavior for nonstructured soils is illustrated as the solid curve in Fig. 1(a). For nonstructured soils, the structural shrinkage phase is absent, and the shrinkage behavior has been widely conceptualized as three graphically distinguishable stages from fully saturated to the completely dry state: (1) normal shrinkage (Cornelis et al. 2006), basic shrinkage (Mitchell 1992), or proportional shrinkage (Groenevelt and Grant 2001); (2) residual shrinkage or transition shrinkage (Braudau et al. 1999; Peng and Horn 2005); and (3) zero shrinkage (Kim et al. 1992; Chertkov 2000; Peng and Horn 2005).

In the saturated state, a decrease in water content will result in an equal decrease in bulk soil volume. Hence the shrinkage curve evolves from the initial void ratio along the diagonal line, and the slope of SSC equals unity (e.g., Mitchell 1992; Chertkov 2000). As a soil continues to dry, air enters the intra-aggregate pores and a further decrease in water content (volume) upon drying exceeds the volume reduction of the voids or bulk soil, initiating the second stage, called residual shrinkage. Here, the slope of SSC or shrinkage rate becomes less than one. With continued drying, the SSC gradually flattens out and the bulk soil reaches its densest configuration. This stage is defined as zero shrinkage because the total volume of soil has been considered unaltered as the water content further decreases (e.g., Bruand and Prost 1987; Bronswijk 1991). In the zero-shrinkage state, although the water content continues to decrease, the volume shrinkage has been considered only to transfer intraparticle space to interparticle layers (e.g., Bruand and Prost 1987). Therefore the shrinkage of micro pores has been considered to compensate for the expansion of macro pores, leading to much less pronounced shrinkage behavior of the total volume.

The current or traditional phenomenologically based interpretation of soil shrinkage behavior causes some inconsistency in terminologies, and the definitions of different stages lack clear physical meanings. Although the shrinkage behavior of soil has been thought to be related to SWR behavior (Fredlund et al. 2002; Peng and Horn 2005; Lin and Cerato 2013), the lack of clear understanding of SWR mechanisms and inaccuracy in SSC measurement techniques, particularly in high suction or low soil water-content regimes, has restricted further development of SSCs.

In light of recent breakthroughs in the understanding of SWR mechanisms (Lu and Khorshidi 2015) and their quantitative linkages to several fundamental SWR characteristics such as SSA (Khorshidi et al. 2017) and CEC (Khorshidi and Lu 2017), and advances in laboratory testing of SSCs (e.g., Dong and Lu 2017), new insights into the behavior of the SSC and its relation to SWR characteristics can be established. A new conceptual model for the SSC, based on SWR mechanisms, is proposed below.

The proposed soil shrinkage behavior is illustrated in Fig. 1(b). The SSC can be conceptualized in two regimes distinguished by capillary and adsorption SWR mechanisms. The capillary regime can be further divided into two states: capillary and pendular, with

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**Fig. 1.** Conceptual illustration of soil shrinkage characteristic curve: (a) traditional interpretation; (b) the proposed interpretation based on adsorption and capillary water-retention regimes
the former dominated by saturation state and the latter by the existence of the curved air–water interfaces. In the capillary state a reduction in water volume will result in the same reduction in bulk soil volume, whereas in the pendular state air enters the soil sample and a reduction in water volume will result in a lesser reduction in bulk soil volume. The boundary between the capillary and pendular states can be drawn as the air-entry point when soil starts to desaturate. The adsorption regime can be further divided into two states: adsorbed and tightly adsorbed. The former is dominated by hydration, such as some silty soils and nonexpansive clays, there would be no or few dehydrated cations in the tightly adsorbed state, leading to zero or nearly zero shrinkage rates. Any soils with strong cation hydration, such as expansive clay, will exhibit a non-zero shrinkage rate, as shown through the experimental program in the following section. Furthermore, it is shown that the shrinkage rate in this state is highly correlated with some SWR characteristics such as SSA, CEC, and SWRC.

### Experimental Program and Data Set

In order to quantify the relation between SSC and SWR characteristics, a comprehensive experimental program is conducted on seven silty and clayey soils. Some of the test results—the geotechnical properties, SSA, CEC, and SWRC—are from some previously published work. Others—the SSC, suction stress characteristic curve (SSCC), and elastic modulus function—are from this study.

### Geotechnical Properties, CEC, SSA, and SWRC Measurements

The geotechnical properties of the seven soils are from Lu and Kaya (2014), and are shown in Table 1. The Unified Soil Classification System (USCS) classification of these soils includes the full range of fine-grained soils: ML, MH, CL, and CH. These soils mainly range from silty soil to clayey soil, and according to McKeen’s expansive soil classification (McKeen 1992) they are nonexpansive clay (Georgia kaolinite), low-expansive silt (e.g., Bonny silt), expansive clay (e.g., Denver claystone), and high-expansive clay (Denver bentonite). Correspondingly, the Atterberg limits of these soils have wide ranges: 25 < LL < 118, 17 < PL < 45, and 4 < PI < 73.

The SSA data are from Khoshidi et al. (2017), and were obtained by using a SWR-based methodology. The method assesses the specific moisture capacity function through the identification of the critical points of monolayer or bilayer water molecule coverage from the measured soil water sorption isotherms or SWRCs for

### Table 1. Geotechnical Index Properties and Soil Classification of the Examined Soils

<table>
<thead>
<tr>
<th>Number</th>
<th>Soil</th>
<th>Expansive classification</th>
<th>USCS</th>
<th>Porosity (n)</th>
<th>Atterberg limits&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bonny silt</td>
<td>Low-expansive</td>
<td>ML</td>
<td>0.417</td>
<td>LL (%) = 25.0, PL (%) = 21.0, PI = 4.0</td>
</tr>
<tr>
<td>2</td>
<td>BALT silt</td>
<td>Low-expansive</td>
<td>ML</td>
<td>0.458</td>
<td>LL (%) = 27.4, PL (%) = 21.7, PI = 5.8</td>
</tr>
<tr>
<td>3</td>
<td>Iowa silt</td>
<td>Low-expansive</td>
<td>ML</td>
<td>0.492</td>
<td>LL (%) = 33.7, PL (%) = 22.4, PI = 11.3</td>
</tr>
<tr>
<td>4</td>
<td>Kaolinite</td>
<td>Nonexpansive</td>
<td>MH</td>
<td>0.522</td>
<td>LL (%) = 44.0, PL (%) = 26.0, PI = 18.0</td>
</tr>
<tr>
<td>5</td>
<td>Missouri clay</td>
<td>Expansive</td>
<td>CL</td>
<td>0.490</td>
<td>LL (%) = 36.0, PL (%) = 17.0, PI = 19.0</td>
</tr>
<tr>
<td>6</td>
<td>Claystone</td>
<td>Expansive</td>
<td>CL</td>
<td>0.471</td>
<td>LL (%) = 44.0, PL (%) = 23.0, PI = 21.0</td>
</tr>
<tr>
<td>7</td>
<td>Bentonite</td>
<td>High-expansive</td>
<td>CH</td>
<td>0.692</td>
<td>LL (%) = 118.0, PL (%) = 45.0, PI = 73.0</td>
</tr>
</tbody>
</table>

<sup>a</sup>According to the method by McKeen (1992).

<sup>b</sup>Data from Lu and Kaya (2014).
adsorption and desorption in the high suction range. The method-
ology has been validated against the conventional ethylene glycol
monoethyl ether (EGME) method and other SWR-based methods.
The SSA data are listed in Table 2.

The CEC data are from Khorshidi and Lu (2017), and were ob-
tained by using a SWR model considering the charge-dipole in-
teraction for the cation hydration and experimental data of SWR in the
high suction range. The CEC values obtained from the SWR model
have been compared with those obtained by the conventional am-
monium acetate method and excellent agreement was obtained. The
CEC data are also listed in Table 2.

The SWRC parameters were obtained by fitting the experi-
mental data with the SWRC model of Lu (2016). The SWR data of all
seven soils are from previously published work. Among them, the
SWR data for Bonny silt, Georgia kaolinite, and Denver claystone
are from Dong and Lu (2016) for adsorptive water. The SWR data for Bonny silt, Georgia kaolinite, and Denver claystone
are from Khorshidi and Lu (2017), and the rest are from Dong and Lu
(2016). Because high matric suction is involved in the adsorption
regime, more than one measurement technique was involved in
obtaining the SWR data for the seven soils. All SWRCs were ob-
tained by combining the results from the transient water release and
imbibition method (TRIM) (Wayllace and Lu 2012) for capillary
water and the results from vapor sorption isotherm (Likos et al.
2011) for adsorptive water. The SWR data for selected soils—
Bonny silt, Georgia kaolinite, Denver claystone, and Denver
bentonite—are shown in Fig. 2. The SWRC model of Lu (2016)
explicitly and quantitatively distinguishes capillary water from
adsorptive water. For completeness, a brief description is provided
here. The total volumetric water content \( \theta_a(\psi) \) can be expressed as the sum of adsorptive water content \( \theta_a(\psi) \) and capillary water
content \( \theta_c(\psi) \) in equilibrium with the prevailing matric suction \( \psi \), quantified by the following expressions:

\[
\theta_a(\psi) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{\psi - \psi_{\text{cav}}}{0.25 \psi_{\text{cav}} \sqrt{2}} \right) \right] \left[ \theta_a(\psi) - \theta_c(\psi) \right] \{ 1 + c \psi \}^{1/N-1}
\]

where \( \alpha \) and \( N = \) inverse of air-entry suction and the pore size
distribution parameter, respectively; and \( \psi_{\text{cav}} = \) mean cavitation
suction. Eqs. (1)–(3) provide a quantitative assessment of the
adsorptive water and capillary water with seven physically mean-
ningful parameters. The SWRC parameters identified by fitting
Eqs. (1)–(3) with the experimental data are listed in Table 2,
and the corresponding SWRCs are shown in Fig. 2.

SSCC Measurement and Calculation
Suction stress at varying water contents or moisture ratios was mea-
sured by using a drying cake technique (Lu and Kaya 2013; Dong
and Lu 2017). An environmental chamber with an integrated loading
frame and particle image velocimetry (PIV) system was designed
to measure the Young’s or elastic modulus with a mini
loading actuator, and to measure the deformation of the soil cake
induced by drying with a digital still camera. A digital balance with
0.01-g accuracy was used to continuously monitor the weight
change of the soil sample during the drying process. Suction stress
clace \( \Delta \sigma^s \) between two sequential water contents can be cal-
culated as (Lu and Kaya 2013)

\[
\Delta \sigma^s(S) = \frac{E(S)}{(1-\nu) \cdot r(S)} \Delta u_r(r,S) + \frac{E(S) \cdot u_r(r,S)}{(1-\nu) \cdot r^2(S)} \Delta r
\]

where \( E(\theta) = \) elastic modulus function; \( u_r = \) radial displacement;
\( r = \) distance to the displacement field center; and \( \nu = \) Poisson ratio.
A constant Poisson’s ratio of 0.25 is assumed for all soils (Lu and
Kaya 2013).

Measurements of the elastic modulus function \( E(\theta) \), radial dis-
placement \( u_r(\theta, r) \), and displacement center \( r(\theta) \) are independent of
each other but can be conducted concurrently at the same water
content. The loading actuator applied a vertical stress up to 100 kPa
with a small speed (<0.25 \( \mu m/s \)) in order to ensure generation of
reversible elastic deformation and uniform redistribution of the soil
moisture inside the soil cake. Measurement of elastic modulus
function was conducted by the mini loading system to record
the vertical stress and strain (see Dong and Lu 2017 for more de-
tails). The results of the measured elastic modulus data for selective
soils are shown in Fig. 3. The moisture ratio decreased during the
Table 2. SWR Parameters by Lu’s (2016) Model, Surface Properties, Volume Shrinkage Rate in Adsorptive Water-Retention Regime, and Power Law Parameter of Unsaturated Elastic Modulus Function by Lu and Kaya’s (2014) Model for the Examined Soils

<table>
<thead>
<tr>
<th>Number</th>
<th>Soil</th>
<th>Parameters of SWRC model&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Surface properties</th>
<th>Shrinkage rate in adsorption regime</th>
<th>Power of modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bonny silt</td>
<td>( A = 0.019 ) ( N = 1.53 ) ( \theta_a = 0.417 ) ( \theta_{\text{a}}^{\text{max}} = 0.024 )</td>
<td>( \text{SSA} = 111.94 ) ( \text{CEC} = 0.21 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>2</td>
<td>BALT silt</td>
<td>( A = 0.059 ) ( N = 1.72 ) ( \theta_a = 0.458 ) ( \theta_{\text{a}}^{\text{max}} = 0.024 )</td>
<td>( \text{SSA} = 76.16 ) ( \text{CEC} = 0.24 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>3</td>
<td>Iowa silt</td>
<td>( A = 0.083 ) ( N = 1.55 ) ( \theta_a = 0.492 ) ( \theta_{\text{a}}^{\text{max}} = 0.046 )</td>
<td>( \text{SSA} = 104.09 ) ( \text{CEC} = 0.22 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>4</td>
<td>Kaolinite</td>
<td>( A = 0.004 ) ( N = 1.86 ) ( \theta_a = 0.522 ) ( \theta_{\text{a}}^{\text{max}} = 0.009 )</td>
<td>( \text{SSA} = 28.26 ) ( \text{CEC} = 0.09 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>5</td>
<td>Missouri clay</td>
<td>( A = 0.022 ) ( N = 1.57 ) ( \theta_a = 0.490 ) ( \theta_{\text{a}}^{\text{max}} = 0.081 )</td>
<td>( \text{SSA} = N/A ) ( \text{CEC} = N/A )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>6</td>
<td>Claystone</td>
<td>( A = 0.010 ) ( N = 1.56 ) ( \theta_a = 0.471 ) ( \theta_{\text{a}}^{\text{max}} = 0.111 )</td>
<td>( \text{SSA} = 152.64 ) ( \text{CEC} = 0.35 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
<tr>
<td>7</td>
<td>Bentonite</td>
<td>( A = 0.014 ) ( N = 1.41 ) ( \theta_a = 0.692 ) ( \theta_{\text{a}}^{\text{max}} = 0.156 )</td>
<td>( \text{SSA} = 659.72 ) ( \text{CEC} = 1.69 )</td>
<td>( \theta_a - \theta_c )</td>
<td>( \rho^2 )</td>
</tr>
</tbody>
</table>

<sup>a</sup>Lu’s model of SWR (Lu 2016).
<sup>b</sup>Data from Khorsheid and Lu (2017).
<sup>c</sup>Data from Khorsheid et al. (2017).
<sup>d</sup>Lu and Kaya’s model of elastic modulus (Lu and Kaya 2014).
drying process, and the elastic modulus for most soils gradually increased.

The radial displacement field and displacement center were obtained from digital still camera images and processed by PIV software (White et al. 2003). The image of the cake was discretized into a mesh to obtain the displacement vector field and displacement center as the sample dried. The SSCCs were obtained by Eq. (4) using the measured elastic modulus function, and the calculated radial displacement field and displacement center are shown in Fig. 4. Details of the theoretical development and experimental procedure can be found in Lu and Kaya (2013).

SSC Measurement, Result, and Calculation

In order to accurately measure and compute the total void ratio reduction and quantitatively differentiate the void ratio reduction due to suction stress reduction and elastic modulus increase during the shrinkage process, the following power law for the elastic modulus function (Lu and Kaya 2014) was used to fit the measured experimental data:

\[
\frac{E_{\text{dry}} - E}{E_{\text{dry}} - E_{\text{wet}}} = \left( \frac{\theta - \theta_{\text{dry}}}{\theta_{\text{wet}} - \theta_{\text{dry}}} \right)^p
\]

where \(E_{\text{dry}}\) and \(E_{\text{wet}}\) = elastic modulus at a dry (\(\theta_{\text{dry}}\)) and a wet (\(\theta_{\text{wet}}\)) state, respectively; and \(p\) = power law fitting parameter. In this study, the wet state is the fully saturated state, and the dry state is the driest state under the test room relative humidity (~15%). A least-squares regression was used to fit Eq. (5) with the measured elastic modulus data, and the resulting \(p\) values for all seven soils are listed in Table 2. Fig. 3 illustrates the comparisons between the measured modulus data and the fitted power law for Bonny silt, Georgia kaolinite, Denver claystone, and Denver bentonite.

The void ratios at varying water contents or moisture ratios were measured and computed using a drying cake technique (Dong and Lu 2017). All soil samples were prepared in thin soil cakes with initial dimensions of 7.6 cm diameter and ~1.7 cm thickness. With the help of the image processing and loading system, the changes in diameter and thickness of the cake sample during the drying process were measured. Fig. 5 presents the experimental results of four typical types of soils: low-expansive silt (Bonny silt), nonexpansive clay (Georgia kaolinite), expansive clay (Denver claystone), and high-expansive clay (Denver bentonite). By comparison of the void ratio changes for these four different types of soils, the shrinkage range varied greatly, depending on soil type. Silty soil shrank from a void ratio of 0.72 to 0.55, whereas expansive clay shrank from 0.89 to 0.42 for Denver claystone and from 2.23 to 0.75 for Denver bentonite.

The extent of shrinkage also varied at different soil water regimes for different soils. For nonexpansive clay, Georgia kaolinite shrank almost 95% of the entire volume reduction in the capillary stages. However, for silty soil, Bonny silt attained 47% of the shrinkage before the air entered the bulk soil. For expansive clay, Denver claystone only decreased a similar 40% of the total volume shrinkage before the shrinkage curve deviated from the diagonal line. For high-expansive soil, Denver bentonite only attained 22% of the total shrinkage within the capillary regime. After the capillary and pendular stages, silty soils do not shrink much in
the adsorption water regime due to lack of clay content. Therefore the shrinkage rate is close to zero. By contrast, expansive clays still retain 25–35% of their shrinkage capacity after the soils enter the adsorption water-retention regimes.

The slope of the shrinkage curve or shrinkage rate is calculated by $\partial e / \partial m$, and is also presented in Fig. 5. Denver claystone and Denver bentonite clearly show high shrinkage rates in the range of adsorption regime (e.g., ~0.17 for claystone and ~0.27 for

**Fig. 3.** Measured and fitted elastic modulus as a function of moisture ratio for (a) Bonny silt; (b) Georgia kaolinite; (c) Denver claystone; (d) Denver bentonite

**Fig. 4.** Measured suction stress characteristic curves as a function of moisture ratio for (a) Bonny silt; (b) Georgia kaolinite; (c) Denver claystone; (d) Denver bentonite
bentonite, but only ~0.09 for Bonny silt). Georgia kaolinite shows an exceptional result: the volume first shrinks until the end of the capillary regime and then swells as the soil further dries toward the dry end. Hence the shrinkage rate becomes negative (~ −0.09) in the adsorption regime. The swelling behavior of Georgia kaolinite is consistent with several previous studies (Vesga 2009; Lu and Kaya 2013).

As the matric suction increases, the volumetric water content and void ratio of the soil decreases. The distinction of capillary water and adsorption water by using Lu (2016) model of SWRC proves that the reduction of void ratio consists of two phases, dominated by capillary water and adsorption water, respectively. It can be seen that expansive clays with greater adsorption water content result in greater reduction of total volume (Figs. 2 and 5). In contrast, silty soil and nonexpansive clay with small adsorption water content show either trivial volume shrinkage or slight expansion (Figs. 2 and 5). The slight expansion exhibited by Georgia kaolinite has been previously interpreted as the result of a lack of cation hydration and the existence of capillary water among submicron particles (Vesga 2009; Lu and Kaya 2013).

From the perspective of mechanics, displacement of a soil cake during a drying process is caused by two mechanisms—increase in stiffness (elastic modulus) and decrease in suction stress—and can be divided accordingly into two superimposed components. Therefore changes in the total void ratio $\Delta e$ can be expressed as

$$\Delta e = \Delta e^\sigma + \Delta e^E$$  \hfill (6a)

and the reduction in the void ratio $\Delta e^\sigma$ due to suction stress and the increase in the void ratio $\Delta e^E$ due to elastic modulus are (see Appendix for details)

$$\Delta e^\sigma = 3(2\nu - 1)\left(\frac{1}{E}\Delta\sigma^s\right)$$  \hfill (6b)

$$\Delta e^E = 3(2\nu - 1)\left(-\frac{\sigma^s}{E^2}\Delta E\right)$$  \hfill (6c)

Given an initial void ratio $e_0$, the total void ratio evolution with the moisture ratio due to the increase in elastic modulus, $e^E$, and due to the change of suction stress, $e^\sigma$, can be expressed as (see Appendix for computation procedure)

$$e^\sigma = e_0 - \sum_m \Delta e^\sigma$$  \hfill (7a)

$$e^E = e_0 - \sum_m \Delta e^E$$  \hfill (7b)

Fig. 6 presents the computed total volume shrinkage or void ratio variation as well as its components due to changes in elastic modulus and suction stress.

**Shrinkage Behavior**

**Shrinkage due to Decrease in Suction Stress and Increase in Modulus**

As shown in Fig. 6, the void ratio reduction due to the decrease in suction stress is generally higher than the void ratio reduction due to the increase in elastic modulus. The former is approximately 1.2 to more than 3 times larger than the latter. For nonexpansive soils of
Bonny silt [Fig. 6(a)] and Georgia kaolinite [Fig. 6(b)], the ratio of suction stress–induced shrinkage to elastic modulus–induced shrinkage was higher in the capillary regime than that in the adsorption regime, whereas for expansive soils of Denver claystone [Fig. 6(c)] and Denver bentonite [Fig. 6(d)], this shrinkage ratio was similar in magnitude in both capillary and adsorption regimes (see Fig. 5 for delineation of capillary and adsorption regimes for these soils). The higher shrinkage amount of expansive clays is due to the higher change in the elastic modulus with water content than that for the nonexpansive soils, as shown in Fig. 3. The computed SSCs (shown as solid curves in Fig. 6) were also compared with the measured SSC (shown as hollow squares). In general, the SSCs determined by the two different methods matched quite well, indicating the validity and accuracy of the computational procedures outlined through Eqs. (6) and (7) for quantitative separation of shrinkage due to the two mechanical mechanisms.

**Correlation between SSC and SWR Characteristics**

Given the shrinkage rate shown in Fig. 5, further investigation was conducted on the correlation of volumetric shrinkage rate in adsorption water retention regime with different SWR characteristics—namely, the specific surface area [Fig. 7(a)], the cation exchange capacity [Fig. 7(b)], and the maximum adsorption water content [Fig. 7(c)]—for all examined soils. The shrinkage rate in the adsorption regime increased as the specific surface area increased, and also increased as the cation exchange capacity increased for different soils. This indicates that in the adsorption water-retention regime the particle hydration or adsorption of water molecules, including surface hydration and cation hydration, is the dominating mechanism causing shrinkage.

For soils with small SSA and CEC, effective hydration is minimal, and the total volume can remain unchanged or even expand as the soil continues to dry into the adsorption regime. This was particularly evident in the result of Georgia kaolinite, in which the very subtle effect of surface hydration did not provide much capability for shrinkage after the capillary state gradually diminished over the cavitation threshold along the drying path [Figs. 5(b) and 6(b)]. Therefore, as the capillary water diminishes, the capillary force at particle contacts releases the soil skeleton without enough constraint provided by physicochemical forces and results in a slight volume swell (Vesga 2009; Lu and Kaya 2013).

For soils with moderate SSA (~100 m²/g) and CEC (~0.23 meq/g) values, such as low-expansive silts (Bonny silt, BALT silt, and Iowa silt), the water desorbed from the moderate surface area of the soil particles and dehydration of cations is subtle [Fig. 5(a) for Bonny silt]. Hence the volume shrinkage in the adsorption water-retention regime is minor and the shrinkage rate is small [less than 0.1, e.g., Bonny silt shown in Fig. 5(a)].

For saturated expansive clays with higher SSA and CEC values, such as Denver claystone and Denver bentonite, initial void ratios are usually high, indicating large interlayer space within the particles. The high SSA and exchangeable cations provide large capacity for shrinkage and considerable shrinkage potentials in the adsorption regime. Therefore the volume shrinkage in the adsorption water retention regime is prominent and the shrinkage rate is higher than 0.15 [Figs. 5(c and d)].

The trendslines of these three groups of correlations show good agreement in a logarithmic relationship, with the coefficient of correlation $R^2$ higher than 0.87. Considering the different intercepts or constant numbers in each trendline functions of correlation, which vary depending on specific physical properties, the three equations

Shrinkage rate, \( \frac{\theta}{\theta_{\text{a}}} \) -0.1 0.0 0.1 0.2 0.3

Maximum adsorption water content

![Graph (a)](image)

Specific surface area, SSA (m²/g)

![Graph (b)](image)

Cation exchange capacity, CEC [meq/g]

![Graph (c)](image)

Maximum adsorption water content

**Fig. 7.** Correlation of the shrinkage rate in adsorption water regime with (a) specific surface area (SSA); (b) cation exchange capacity (CEC); (c) maximum adsorption water content [(a and b) data from Khorshidi and Lu 2017; Khorshidi et al. 2017]

shown in Fig. 7 are found to share the same proportional values for the shrinkage rate to the logarithm of each property [i.e., \( m = 0.12 \ln(x) \) where \( x \) could be SSA, CEC, or \( \theta_{\text{a}} \)]. This implies that linear relations exist among SSA, CEC, and \( \theta_{\text{a}} \). The linear correlation between SSA and CEC has been observed in some previous studies (e.g., Cerato 2001).

**Practical Implications of Strong Correlation between SSC and SWR Characteristics**

The finding of strong correlation between SSC and SWR characteristics bears several important implications for geotechnical engineering research and practices. First, this paper reveals unambiguously for the first time that the shrinkage rate is not zero for silty and clayey soil, implying that soil under field drying conditions can continue to shrink at low water content. Although the shrinkage rate is relatively smaller in the adsorption regime than in the capillary regime, suction stress reduction is much higher, typically over an order of magnitude greater than in the capillary regime. Experimental evidence reported here indicates that suction stress can reach several hundred kPa when soil is dry, implying that it can provide a strong source for either soil stiffness enhancement or crack development under field drying conditions.

The finding of strong quantitative correlations between the shrinkage rate and SWR characteristics provides potential ways to utilize SSC. Quantifications of SSA, CEC, and total amount of adsorptive water are important for much geotechnical and geo-environmental research and practice. However, the common or standardized techniques for measuring these quantities typically are labor-intensive, expensive, and inaccurate for their extensive use of nonwater adsorbates such as EGME and ammonium. Upon further confirmation and generalization, it is possible to use shrinkage rate for the quantification of a soil’s SSA, CEC, and total amount of adsorptive water.

The findings of strong quantitative correlation provide new ways to classify expansive soil. Identifying, quantifying, and classifying expansive soil are part of geotechnical practice in areas where expansive soil is present. Current methodologies, such as geotechnical index—based and swelling potential—based methodologies, are primarily empirical. Some recent advances in expansive soil classifications (e.g., Akin and Likos 2014) are SWR-based. In light of the findings here, further improvement or new classification methodologies based on SSC could be developed.

The finding of the shrinkage rate in the adsorption regime follows the same logarithmic form of SSA, CEC, and maximum adsorptive water content for all tested soils, implying that SSA, CEC, and maximum adsorptive water content are possibly linearly correlated. Therefore it is possible to quantify one parameter from the others.

**Summary and Conclusions**

A critical examination of soil-shrinkage behavior was provided by using comprehensive experimental evidence from different and independent measurements of geotechnical index properties SSA, CEC, SWRC, SSCC, and SSC for various silty and clayey soils. This paper demonstrated experimentally that the traditional interpretations of SSC are incorrect when soil approaches the dry state. The SSC is traditionally divided into three characteristic states: normal shrinkage when soil is saturated, residual shrinkage when soil is unsaturated and subjected to volume reduction, and zero shrinkage when soil approaches the dry state. This paper showed experimentally that shrinkage rate is not zero for silty and clayey soil when soil approaches the dry state. A new conceptual model for SSC was proposed in which SSC can be divided into four distinctive states: capillary, pendular, adsorbed, and tightly adsorbed; each state is governed by one or/and two SWR mechanisms: capillary and adsorption.

Two mechanical mechanisms are responsible for soil shrinkage during a free-confining drying condition: increase in elastic modulus and decrease in suction stress. This paper showed experimentally that void ratio reduction due to the reduction in suction stress is generally higher than that due to the increase in elastic modulus, ranging from 1.2 to more than 3 times greater. For nonexpansive soils, the ratio of suction stress–induced shrinkage to elastic modulus–induced shrinkage is higher in the capillary regime than in the adsorption regime, whereas for expansive soils, this shrinkage ratio is similar in magnitude in both capillary and adsorption regimes.

Most importantly, this paper showed that the shrinkage rate in the adsorption regime is highly correlated with SWR characteristics, namely, the specific surface area, CEC, and total amount of adsorptive water. Each of these three SWR characteristics follows the same exponential form with the shrinkage rate. A correlation coefficient of 0.95 was found between the shrinkage rate and the specific surface area, a correlation coefficient of 0.87 between...
the shrinkage rate and the cation-exchange capacity, and a correlation coefficient of 0.89 between the shrinkage rate and the maximum adsorptive water content. The fact that the shrinkage rate follows the same logarithmic form for each of the three SWR characteristics implies that linear relations or constant scalars exist among a soil’s specific surface area, CEC, and maximum adsorption water content. By linking SWRC/SSCC and Atterberg limits, the SSC or the shrinkage rate shows better quantification of the shrinking/swelling capability and potential application in classifying expansive soils.

Appendix. Computation of Variations of Void Ratio and Its Components

The total volumetric strain of a drying soil cake can be defined by

\[ \varepsilon_v = \frac{\Delta V}{V} \]  

(8)

The relationship between the volumetric strain and void ratio can be derived as

\[ \Delta \varepsilon = \varepsilon_v / (1 - n) \]  

(9)

Assuming that the soil moisture distribution throughout the drying process is homogeneous within a soil cake, the strain in all directions is equal, and the suction stress \( \sigma^* \) developed within the soil cake in all directions is equal as well, i.e.

\[ \varepsilon_v = 3 \varepsilon_z = 3 \varepsilon_r \]  

(10)

\[ \sigma^* = \sigma_z = \sigma_r \]  

(11)

where \( \varepsilon_z \) and \( \varepsilon_r \) is vertical and radial strain, respectively; and \( \sigma_z \) and \( \sigma_r \) is vertical and radial stress, respectively.

According to the two-dimensional theory of elasticity (Lu and Kaya 2013), the stress and strain relationship of the soil cake can be expressed as

\[ \varepsilon = \frac{\sigma_z}{E} - \nu \frac{\sigma_r}{E} = \frac{\sigma_z}{E} - 2\nu \frac{\sigma_r}{E} = \sigma^* - \frac{2\nu - 1}{E} \]  

(12)

\[ \varepsilon_v = 3 \sigma^* - \frac{2\nu - 1}{E} \]  

(13)

where \( \nu \) is Poisson ratio.

An incremental form of the total strain can be expressed as

\[ \Delta \varepsilon_t = 3(2\nu - 1) \left( \frac{1}{E} \Delta \sigma^* - \frac{2\nu - 1}{E} \Delta \varepsilon_v \right) = \Delta \varepsilon^* + \Delta \varepsilon^E \]  

(14)

where \( \Delta \varepsilon^* \) and \( \Delta \varepsilon^E \) is strain component induced by the change of suction stress and change of elastic modulus, respectively. The void ratio variations can be expressed as Eqs. (6a)–(6c).

Given the initial void ratio \( \varepsilon_{v0} \), the void ratio evolves with the moisture ratio due to the change of elastic modulus, \( \varepsilon^E \), and due to the change of suction stress, \( \varepsilon^* \), as the soil cake dries, and can be obtained by integrating Eqs. (6b) and (6c) into Eqs. (7a) and (7b).

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OpenPIV [Computer software]. OpenPIV, Tel Aviv, Israel.


Erratum for “Correlation between Soil-Shrinkage Curve and Water-Retention Characteristics” by Ning Lu and Yi Dong

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In this erratum, we correct three errors made in producing Figs. 4 and 6. Though these errors do not invalidate the major findings of the paper, they do affect the quantitative results in the suction stress and void ratio. Fig. 4 shows the computed suction stress curves from the measured soil shrinkage data, and Fig. 6 shows the results of the computed soil shrinkage curves in comparison with the measured soil shrinkage data in various soils. The three errors are missing the effect of vertical displacement in Eq. (4), missing the effect of the initial void ratio effect in Eqs. (6b) and (6c), and implementing a wrong sign in the void ratio change due to the increase in the elastic modulus during soil shrinkage.

The first error in Eq. (4) was made in the assumption of a plane-stress problem in developing a theory for interpreting the shrinkage tests in Lu and Kaya (2013). For a drying soil specimen with a cylindrical cake geometry, an assumption of a plane-stress problem was made as a first approximation, leading to the neglect of the vertical displacement due to the suction stress. However, even though the surface of the specimen is free of the total stress, suction stress or effective stress exists in every point of the specimen in all three directions. Considering the existence of suction stress in all three directions and following the same solution procedure in Lu and Kaya (2013), a more rigorous

Fig. 4. Comparisons of the calculated suction stress characteristic curves as a function of moisture ratio between the improved Eq. (4) and the original equation for (a) Bonny silt; (b) Georgia kaolinite; (c) Denver claystone, and (d) Denver bentonite.
The equation for the incremental suction stress change \( \Delta \sigma^s \) can be arrived at as follows:

\[
\Delta \sigma^s(\theta) = -\frac{u_r(r, \theta)}{(1 - 2\nu) \cdot r(\theta)} \Delta E(\theta) - \frac{E(\theta)}{(1 - 2\nu) \cdot r(\theta)} \Delta u_r(r, \theta) + E(S) \cdot u_r(r, \theta) \Delta r
\]

where \( E(\theta) \) = elastic modulus as a function of volumetric water content \( \theta \); \( u_r \) = radial displacement; \( r \) = distance to the displacement field center; and \( \nu \) = Poisson’s ratio.

In comparison with the original equation, a factor of 2 is added in front of Poisson’s ratio in Eq. (4). With this improvement, the suction stress characteristic curves (SSCCs) for the various soil shrinkage tests can be calculated and replotted with comparisons to the suction stress characteristic curves in the original Fig. 4.

As shown, assuming 0.25 of the Poisson’s ratio, about 50% of relative error between the revised SSCC and the original one can be observed.

The second error involves missing one term of the initial void ratio effect in converting the void ratio from volumetric strain. Under the suction stress–based effective stress framework, soil shrinkage curve can be computed as follows:

\[
\Delta e_t = \Delta e^s + \Delta e^E
\]

\[
\Delta e^s = 3(2\nu - 1)(1 + e_0) \left( \frac{1}{E} \Delta \sigma^s \right)
\]

\[
\Delta e^E = 3(2\nu - 1)(1 + e_0) \left( -\frac{\sigma^s}{E} \Delta E \right)
\]

Fig. 6. Total shrinkage void ratio and its components due to the change in elastic modulus and suction stress for different soils: (a) Bonny silt; (b) Georgia kaolinite; (c) Denver claystone; and (d) Denver bentonite.
\[
e^e = e_0 - \sum_m \Delta e^m \quad (7a) \\
e^E = e_0 - \sum_m \Delta e^E \quad (7b)
\]

where \(\Delta e^t\) = total void ratio reduction; \(\Delta e^e\) = void ratio reduction due to suction stress decrease; \(\Delta e^E\) = increase in void ratio due to elastic modulus hardening; and \(e_0\) = initial void ratio. Eqs. (6b) and (6c) correct the missing factor of \((1 + e_0)\) in the calculation of the original paper.

The third error was made when computing the void ratio change due to the increase in the elastic modulus during drying. The term in Eq. (6c) should be negative in value, indicating an increase in void ratio due to soil hardening during drying. The original paper implemented this term as positive in value, leading to an incorrect plot in Fig. 6.

The revised Fig. 6 shows comparisons between the measured soil shrinkage data and the computed soil shrinkage curve after the correction of the two errors. It demonstrates that soil shrinkage during drying is due to two mechanisms with opposite effects: decreasing in suction stress further reduces the void ratio [Eq. (7a)], whereas modulus hardening further increases the void ratio [Eq. (7b)]. It also shows that suction stress–based effective stress can accurately predict soil shrinkage curve.

References