INTRODUCTION

A numerical analysis technique, called the distinct element method, has been applied in the past for an effective analysis of the mechanical behavior of assemblies of discrete particles. The method, originally developed by Cundall (1971) for solving rock mechanics problems, was applied subsequently to analyze the behavior of assemblies of disks (Cundall and Strack 1979; Cundall et al. 1982; Petrakis and Dobry 1988; Ting et al. 1986; Trent 1988) and spheres (Cundall and Strack 1979), and to solve ice mechanics (Mustoc et al. 1989) and biomechanics problems (Williams et al. 1987). It is shown in this note that this can also be used to determine the load-deflection behavior of structures, simulating the axial, shear, and bending deformation modes properly. The results determined by the distinct element method are compared with those obtained by the finite element method (Zienkiewicz 1977).

The distinct element method may potentially be applied to study the mechanical behavior of an assembly of platy particles such as clay particles. It is important to simulate the bending of clay particles in such studies. The study presented herein shows the potential of achieving this using the distinct element method. Furthermore, interaction between clay particles during a loading is complex, involving interparticle sliding, deletion of existing contacts, and formation of new contacts. These mechanical processes cannot be modeled using the standard finite element method, but the distinct element method offers a natural means of simulating such events. Contrary to the standard finite element method, the distinct element method does not involve the formation of a global stiffness matrix. However, the distinct element method requires greater central processing unit (CPU) time.

THEORY

Principles

For details of the basic principles of the distinct element method (DEM), the reader is referred to previously published references [e.g., Cundall et al. (1982)]. A brief summary is presented herein. The system to be analyzed is considered to consist of a set of rigid, discrete particles or distinct elements. The mechanical behavior is modeled as a dynamic process, even when the loading is static. The equation governing the dynamic behavior of a distinct element is solved numerically by an explicit time integration procedure. The method is based on the idea that the time step may be chosen small, such that a disturbance that originates at a given point on the particle...
does not propagate more than one particle distance. Therefore, during a given
time step, a particle will interact only with its immediate neighbors and not
with those beyond them.

When a particle becomes in contact with another during a given time step,
a force develops at the contact. The direction and magnitude of this contact
force depend on the relative displacement between the particles at the contact
point. A suitable force-displacement law is used to compute the contact force.
The changes in forces at existing contacts are also calculated in a similar
manner. Once all the contact forces are calculated for a given particle, its
acceleration is determined by applying Newton's second law, and its velocity
is calculated by integrating the acceleration. Having done this for every par-
ticle in the assembly, relative velocities at the contacts are determined and
integrated to determine the relative displacements, and new set of contact
forces are computed. This calculation cycle is repeated for every time step.

In order to model the dissipation of energy, Cundall et al. (1982) have
introduced two types of damping: global damping and contact damping. The
global damping force was assumed to be proportional to the absolute velocity
of the particle whereas the contact damping force is assumed to be propor-
tional to the relative velocity at the contact. Similar to the Rayleigh damping
model (Clough and Penzien 1975), the damping factor is assumed to be
proportional to an appropriate measure of the stiffness and/or mass.

**Application to Structural Analysis**

In order to define the geometrical and material characteristics necessary
for an analysis by DEM, let us examine a typical plane frame structure shown
in Fig. 1. The structure consists of two straight members AB and CD joined
at D by a rigid connection. The end B is connected to a rigid support $S_1$
through a hinge. The ends A and C are resting on supports $S_2$ and $S_3$, re-
spectively, through rollers. The shear and Young's moduli of the members
are $G$ and $E$, respectively. The second moment of inertia and the rotational
mass moment of inertia of the members are $I$ and $J$, respectively, and the
area is $a$. The mass density of the material of the members is $\rho$. Let us say
that the support $G$ is moved vertically down by a certain distance and the
corresponding internal forces and the deformed configuration are to be determined.

In applying DEM to model the behavior of the structure shown in Fig. 1, which consists of members (e.g., AB and CD) that are continuous, suitable definitions of a distinct element and the manner in which they are in contact with each other or with the supports are needed.

In the present approach, each member is divided into a number of distinct elements (i.e., discretized). This will allow the deformation of the members to be modeled properly. Sufficient number of distinct elements are used to accurately model the elastic deformation of the members. Each distinct element is assumed to be rigid. A typical discretization of the structure shown in Fig. 1 is shown in Fig. 2, where the members AB and CD are divided into six and five distinct elements of equal lengths, respectively. The distinct elements within the members AB and CD are interconnected to each other.

In defining the contact force-displacement relations, we identify two different types of contacts in the structure. The first type, referred to here as the internal contact, is the contact between distinct elements on a given member of the structure (e.g., F, H, etc. in Fig. 2). The second type, referred to here as the edge-to-face contact, is the contact between an end of a distinct element and the face of another distinct element (e.g., D in Fig. 2) or the face of a support (e.g., A, B, and C in Fig. 2).

Three degrees of freedom are defined at every contact. In calculating the contact forces from the relative displacements, suitable definitions of local coordinate systems are necessary. The axial, transverse, and rotational directions are chosen at internal contacts as shown in Fig. 3(a). A normal, shear, and rotational directions are chosen at edge-to-face contacts as shown in Fig. 3(b).

During a given time step, assuming that a disturbance that originates at an internal contact (say, contact 1) of a distinct element propagates only up to the other end (say, contact 2) of this distinct element, the forces at contact 1 are calculated by assuming that contact 2 is fixed. The relative displacements and forces with respect to the local coordinate system are related in this case by
where $K_{11} = Ea/l$, $K_{22} = 12EI/l^3$, $K_{23} = -6EI/l^2$, and $K_{33} = 4EI/l$, with $l$ denoting the length of the distinct element. $\Delta F_n$, $\Delta F_s$, and $\Delta F_t$ are the incremental forces in $n$-, $s$- and $t$-directions corresponding to the incremental relative displacements $\Delta u_n$, $\Delta u_s$, and $\Delta u_t$, respectively. Note that the aforementioned relation corresponds to the case with negligible shear deformation. The reader may refer to standard texts on structural analysis for further details on this relation.

Similarly, a set of force-displacement laws are to be employed in calculating the edge-to-face contact forces. Here, we assume that the normal, shear, and rotational laws are uncoupled; i.e., similar to the one given by (1) with $K_{23} = 0$. In simulating a fixed connection such as contact $D$ in Fig. 2, large values are assigned to the coefficient of stiffness in all three directions. A hinge is modeled by assigning a zero rotational stiffness and a roller by assigning zero shear and rotational stiffness.

The incremental contact forces during a given time step are added on to the forces existing at the beginning of the time step. The forces are then resolved to compute the total forces $(F_x, F_y, F_t)$ in the global $x$-, $y$-, and $t$-directions. In the present analysis, only a global damping is applied. The damping forces in the $x$-, $y$-, and $t$-directions acting on a distinct element are calculated as $F_{cx} = \alpha m v_x$, $F_{cy} = \alpha m v_y$, and $F_{ct} = \alpha m v_t$; where $m = \rho a l$ is the mass of the element; $v_x$, $v_y$, and $v_t$ are velocities of the element in the global directions, and $\alpha$ is a constant of proportionality. A mass-proportional damping, therefore, is employed.
Once the contact and damping forces are computed, the accelerations of a distinct element are calculated by:

\[ a_x = \frac{\sum F_{xi} - F_{ci}}{m} \]  

\[ a_y = \frac{\sum F_{yi} - F_{cy}}{m} \]  

\[ a_t = \frac{\sum F_{ti} - F_{ct}}{J} \]

where \( F_{xi} \) is the \( i \)th contact force in the \( x \)-direction and so on. These are integrated to find new velocities and displacements. The positions of the distinct elements are updated from the knowledge of new displacements, and deformed configuration of the structure is thus determined.

Following the procedure suggested by Cundall et al. (1982), a suitable time step is calculated from the natural frequency of a single degree-of-freedom system whose mass is \( m \) and stiffness is the largest (say, \( K \)) of the stiffness coefficients such as \( K_{11} \) in (1). The time step (\( \Delta t \)) is taken to be \( 1/10 \) of the natural period (i.e., \( \Delta t = \pi/5 \sqrt{m/k} \)).

When solving a static problem, the inertial forces at the end of the analysis must be negligible compared to the static forces. Additionally, the static forces must balance among themselves in order to satisfy the equilibrium condition.

**RESULTS**

The problem shown in Fig. 1 and defined in the preceding section was analyzed by DEM, and the finite element method (FEM) and the results are compared. Some vertical displacements are imposed at contact A (Fig. 2) in both types of analyses. The geometrical and material properties used are \( a = 5 \) sq in., \( I = 10.42 \) in.\(^4\), \( J = 2.1 \) lb-sec\(^2\)-in.\(^4\), \( G = 1.0 \times 10^7 \) lb/sq in., \( E = 2.9 \times 10^7 \) lb/sq in., and \( \rho = 0.2 \) lb-sec\(^2\)/cubic in. The time step is calculated to be 0.00017 sec.

In the analyses by DEM, the support \( S_2 \) was moved with a uniform velocity of \( v_0 \) in the \( y \)-direction (Fig. 4) up to a time \( t_0 \) and held stationary after \( t_0 \). The structure will then perform a free vibration. The analyses were continued up to \( t_1 \) (\( t_1 > t_0 \)). The analyses were repeated for different values of \( v_0 \) and \( \alpha \) and the results are presented in Table 1 and Fig. 4. Also, the analyses were repeated with different lengths of distinct elements, and the discretization shown in Fig. 2 was found to be satisfactory.

Presented in Fig. 4(\( a \)) are the original configuration of the structure and deformed configurations determined by DEM and FEM. The variations of the vertical force at contact A (Fig. 2) and the moment at contact P (Fig. 2) with time according to DEM are presented in Figs. 4(\( b \)) and 4(\( c \)), respectively, for two different values of \( v_0 \) and \( \alpha \). The values predicted by FEM are presented in Table 1. \( t_0 \) is calculated so that final vertical displacement at contact A (i.e., \( t_0 v_0 \)) is the same in all cases. It can be observed that while the use of different values for \( v_0 \) and \( \alpha \) changes the oscillatory nature of the initial transient response, the force and the moment converged close to those predicted by FEM. It takes about 600 cycles for the forces to converge to the static values.

The forces at the supports and the internal contact P predicted by DEM
and FEM are compared in Table 1. The forces at the supports in the x- and y-directions balance very well, indicating that the structure is in equilibrium under the influence of the static contact forces.

With all other conditions remaining the same, the same problem was analyzed with a hinge support (instead of a roller support) at contact C, and the results are compared in Fig. 5. The results show that DEM predicts the elastic curves of the members nicely.

In order to illustrate the ability of DEM to model large rotations and dis-
TABLE 1. Comparison of Results Predicted by DEM and FEM

<table>
<thead>
<tr>
<th>Contact</th>
<th>(F_x) ((1))</th>
<th>(F_y) ((2))</th>
<th>(F_z) ((3))</th>
<th>(F_x) ((4))</th>
<th>(F_y) ((5))</th>
<th>(F_z) ((6))</th>
<th>(F_z) ((7))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>(-5.06 \times 10^4)</td>
<td>0</td>
<td>0</td>
<td>(-5.26 \times 10^4)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>(2.19 \times 10^4)</td>
<td>0</td>
<td>0</td>
<td>(2.42 \times 10^4)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>(2.86 \times 10^4)</td>
<td>0</td>
<td>0</td>
<td>(2.84 \times 10^4)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>(-2.86 \times 10^3)</td>
<td>(6.43 \times 10^5)</td>
<td>0</td>
<td>(-2.83 \times 10^3)</td>
<td>(6.10 \times 10^5)</td>
<td></td>
</tr>
</tbody>
</table>

Original Config.  
Deformed (DEM)  
Deformed (FEM)  
Deformed (DEM)  
Deformed (FEM)

FIG. 5. Results Predicted by DEM and FEM

placements, the support \(S_2\) was moved vertically down through a larger distance and the final, static deformed configuration was predicted and shown in Fig. 5. Reasonable results are obtained. Such large rotations and displacements are likely to occur in an assembly of clay particles subjected to an external load, and it appears that DEM is a suitable method for analysis of such a system.

CONCLUSIONS

The distinct element method is applied in this study to determine the load-deflection behavior of structures. The results predicted by the distinct element method are compared with those predicted by the finite element method and shown to agree reasonably well.

ACKNOWLEDGMENTS

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APPENDIX. REFERENCES


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