Estimating Saturated Hydraulic Conductivity from Surface Ground-Penetrating Radar Monitoring of Infiltration

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Abstract

In this study we used Hydrus-1D to simulate water infiltration from a ring infiltrometer. We generated water content profiles at each time step of infiltration, based on a particular value of the saturated hydraulic conductivity while knowing the other van Genuchten parameters. Water content profiles were converted to dielectric permittivity profiles using the Complex Refractive Index Method relation. We then used the GprMax suite of programs to generate radargrams and to follow the wetting front using arrival time of electromagnetic waves recorded by a Ground-Penetrating Radar (GPR). Theoretically, the depth of the inflection point of the water content profile simulated at any infiltration time step is related to the peak of the reflected amplitude recorded in the corresponding trace in the radargram. We used this relationship to invert the saturated hydraulic conductivity for constant and falling head infiltrations. We present our method on synthetic examples and on two experiments carried out on sand. We further discuss the possibility of estimating two other van Genuchten parameters, \( n \) and \( \alpha \), in addition to the saturated hydraulic conductivity.

1. Introduction

Soil hydraulic properties, represented by the soil water retention \( \theta(h) \) and hydraulic conductivity \( K(h) \) functions, dictate water flow in the vadose zone, as well as partitioning between infiltration and runoff. Their evaluation has important implications for modeling available water resources and for flood forecasting. It is also crucial in evaluating the soil capacity to retain chemical pollutants and in assessing the potential for groundwater pollution.

The determination of parameters defining the van Genuchten soil water retention function (van Genuchten, 1980) is usually done using laboratory experiments, such as the hanging water column (Dane and Hopmans, 2002). On the other hand, the hydraulic conductivity can be estimated either in the laboratory, or \textit{in situ} using infiltration tests. Among the large number of existing infiltration tests (Angulo-Jaramilho, 2002), the single (Müntz et al., 1905) or double ring infiltrometers (Boivin et al., 1987) provide the field saturated hydraulic conductivity by applying a positive pressure on the soil surface, while the disk infiltrometer (Perroux et White, 1988; Clothier and White, 1981) allows for reconstruction of the hydraulic conductivity curve by applying different pressures smaller than or equal to zero. For infiltration tests, the volume of infiltrated water versus time is fitted to infer the soil hydraulic conductivity at or close to saturation. These tests are time-consuming and difficult to apply to landscape-scale forecasting of infiltration. Furthermore, their analysis involves various simplifying assumptions, partly due to the ignorance of the shape of the infiltration bulb.
Geophysical monitoring methods, mainly electrical (Battle-Aguilar et al., 2009) and electromagnetic methods, have been carried out during the infiltration process. Among them, the Ground-Penetrating Radar (GPR) is based on electromagnetic (EM) wave propagation. It is highly sensitive to variations in water contents, which are directly related to the dielectric permittivity (Huismann et al., 2003). This method thus appears to be an accurate tool for monitoring the wetting front movement during infiltration (Saintenoy et al., 2008).

2. Methods

We studied infiltration of a 5-cm thick water layer inside of a single ring infiltrometer in a sandy soil. The schematic of the apparatus is presented in Figure 1. The single ring infiltrometer is a 1-mm thick aluminum cylinder of a 60-cm diameter, approximately 20-cm high, buried in the soil to a depth of 10 cm. We set up a GPR antennae (namely the transmitter T and the receiver R) at a variable distance from the edge of the cylinder, labeled x in Figure 1. In all of our field experiments, we used a Mala RAMAC system with antennae centered on 1600 MHz, shielded at the top. We then covered the inner part of the cylinder with a plastic waterproof sheet. The plastic sheet allowed us to fill the cylinder with water and create an initial 5-cm thick water layer, while preventing infiltration into the sand before starting data acquisition. The beginning of the acquisition was launched by pulling away the plastic sheet to trigger water infiltration. The GPR system was set to acquire a trace every 10 s. With this apparatus, we performed two types of infiltration: i) a falling head infiltration consisting of pulling away the plastic sheet and leaving water to infiltrate into the sand freely with no additional refill, and ii) a constant head infiltration, when water was continuously added to the ring to maintain a 5-cm thick water layer during the infiltration experiment. In the following examples we will show how we can use the GPR data acquired every 10 s during the infiltration experiment to estimate the saturated hydraulic conductivity.

Figure 1. Schematic of the apparatus at its initial state.
3. Falling Head Infiltration

3.1. Numerical Example

3.1.1. Forward Modeling

The falling head infiltration experiment was simulated using Hydrus-1D (Šimůnek et al., 1996, 2008; Šimůnek and van Genuchten, 1996). The soil profile was 50 cm deep, was assumed to be homogeneous, and was divided into 1001 layers. To describe the soil hydraulic properties of the medium, we used the van Genuchten-Mualem (van Genuchten, 1980) hydraulic conductivity and water retention functions, which require 5 parameters, namely the saturated water content, $\theta_s$, the residual water content, $\theta_r$, two fitting parameters, $\alpha$ and $n$, and the saturated hydraulic conductivity, $K_s$. For our numerical example, we set $\theta_s=0.43$, $\theta_r=0.07$, $\alpha=0.019$ cm$^{-1}$, $n=8.67$, and $K_s=0.120$ cm/min. We used an atmospheric boundary condition (BC) with no rain and no evaporation at the soil surface and a free drainage BC at the bottom. To simulate the 5-cm layer of water, the initial condition was set to a 5 cm pressure head in the top node. We simulated the first 10 minutes of the experiment with a time step of 10 s, i.e., with 60 water content snapshots. Using the CRIM relation (Birchak et al., 1974; Roth et al., 1990), each water content snapshot was converted to permittivity profiles (made of 1001 points), considering a three-phase media: sand (considered as pure silica), water, and air. Each one of these permittivity profiles (Fig. 2a) were the input for the GprMax2D program (Giannopoulos, 2004), based on finite difference time domain modeling, the output of which is one simulated GPR trace acquired at the antenna position (set at the surface of the medium, in the middle with $X=0.3$ m in Fig. 1). The simulated GPR monitoring of the infiltration process is shown in Figure 2b. The horizontal axis is the number of traces simulated by GprMax2D, two traces being separated by 10 seconds, as Hydrus-1D profiles are. The vertical axis is the Two-Way Travel time (TWT) of the EM wave amplitude coming back to the receiver.

On the profile presented in Figure 2b, we denote one particular reflection, labeled A. Its arrival time is increasing as the wetting front moves deeper. This reflection is interpreted as coming from the wetting front. The reflections labeled A' and A" are primary and secondary multiples of reflection A. The reflection labeled B is the wave traveling in the air directly between the two antennae. After the 40$^{th}$ trace, the 5-cm layer of water has been infiltrated and drainage is starting. As a consequence, the permittivity of the upper part of the medium decreases. The EM wave velocity is inversely proportional to the square root of permittivity (Huisman et al., 2003). Then the TWT of the reflection A increases more slowly, creating a change of slope in the reflection time curve (Fig. 2b). In Figure 2c, we display two curves: the TWT of the maximum peak of a reflection A (obtained from Fig. 2b) and the TWT calculated by a ray-path algorithm, going from the GPR antennae to the inflection point of $\varepsilon(z)$ curves (indicated by crosses in Fig. 2a), as proposed by Saintenoy and Hopmans (2011). We attribute the difference between these two curves to some numerical dispersion of the signal, a problem that we will address later.
3.1.2. Inversion

We used the TWT obtained from the radargram of Figure 2b as data to be fitted to derive the saturated hydraulic conductivity, assuming the other 4 parameters are known. Using Hydrus-1D, we generated 60 water content snapshots using the saturated hydraulic conductivity in the range from 0.01 to 1 cm/min, with a step of 0.001 cm/min. For each value of $K_s$, we calculated the TWT by the ray-path algorithm from the surface to the inflection point of the $\varepsilon(z)$ curves. We computed the Root Mean Square Error (RMSE) between these times and the data. The RMSE was minimized for $K_s=0.129$ cm/min, which is higher than the value used for simulating the data, i.e., $K_s=0.120$ cm/min.

![Image](image-url)

Figure 2. Falling head infiltration from a 5-cm thick water layer. a) Permittivity profiles: each curve is plotted every 10 s; crosses represent the inflection points of each curve. b) Radargram simulated with GprMax2D; reflection A is coming from the wetting front, B is the direct wave, A' and A'' are multiples of reflection A. c) TWT computed with an optical path algorithm, directly from the permittivity profiles.

3.2. Experimental Example

3.2.1. Experimental Data and its Analysis

The experiment took place in a quarry of Fontainebleau sand in Cernay-La-Ville (Yvelines, France). The middle of the antennae was positioned 11 cm away from the cylinder wall (x = 11 cm in Fig. 1). The 5-cm water layer was fully infiltrated after about 10 minutes, although in certain areas of the soil surface this time has been slightly shorter. The recorded GPR data are shown in Figure 3. We recorded during 30 minutes, with a time window of 15 ns, transmitting
and receiving each 10 seconds (stacking 4 measurements). We subtracted the average trace and applied an Automatic Gain Control to the data. The sand parameters have been determined in laboratory by several classical hanging water column experiments, fitted by the van Genuchten retention curve. Assuming a 5% uncertainty of the optimized parameters, we obtained $\theta_r=0.062 \pm 0.003$, $\theta_s=0.39\pm0.01$, $\alpha=0.023\pm0.001$, and $n=6.7\pm0.3$. We considered in our model that the sand was homogeneous. Gravimetric measurements on field samples gave us an initial volumetric water content of $\theta_i=0.09\pm0.01$. In the profile presented in Figure 3, we denote three particular reflections. The one interpreted as coming from the infiltration front, labeled A, is visible during the first 30 minutes of the acquisition, with an arrival time varying from 2 ns down to 9 ns. The other reflections are interpreted in Léger and Saintenoy (2012). We determined the arrival time of the A reflection peak and inverted the saturated hydraulic conductivity using the same algorithm as for the synthetic case. We obtained the minimum of the objective function for $K_s=0.131$ cm/min. In parallel, we also carried out disk infiltrometer experiments using the multi-potential method (Ankeny et al., 1991; Reynolds and Elrick, 1991). We obtained a value of the saturated hydraulic conductivity of $K_{Disk}=0.108\pm0.01$ cm/min.

![Figure 3. Experimental GPR data acquired during the falling head infiltration (from a 5 cm initial water layer). Reflection A is the reflection coming from the wetting front.](image)

### 3.2.2. Uncertainty Analysis

We assumed a 5% relative uncertainty of four van Genuchten parameters and the initial water content of the sand. These parameters directly influence the arrival time curves used to compute the misfit function. Figure 4 shows variations associated with each of these parameters. It appears that the uncertainty in $\alpha$ has the strongest influence on the arrival time curves. As an attempt to evaluate the uncertainty in the saturated hydraulic conductivity retrieved from GPR data fitting, we made some quadratic error summation associated with each of the parameters. The total quadratic error has the expression of $\delta_{\text{obj}} = \sqrt{\delta_{\text{gpr}}^2 + \delta_{\text{ng}}^2 + \delta_{\text{gs}}^2 + \delta_{\text{gs}}^2 + \delta_{\text{A}}^2 + \delta_{\text{gpr}}^2 + \delta_{\text{gs}}^2}$, where $\delta_{\text{ng}}$, $\delta_{\text{gs}}$, $\delta_{\text{gs}}$, and $\delta_{\text{A}}$ are RMSE due to uncertainties in $\theta_i$, $\theta_r$, $\theta_s$, $\alpha$, and $n$. The RMSE is the summation of the two curves for each parameter, presented in Figure 4. $\delta_{\text{gpr}}=0.1$ ns is assumed to be the error on picking, and $\delta_{\text{A}}=0.01$ ns is the RMSE between the arrival times generated by the optical path-algorithm and the GprMax modeling (Fig. 2c). The total quadratic error was
estimated at $\delta_{\text{tot}}=0.101$ ns. From the objective function curve, all $K_s$ in the interval $[0.04; 0.263 \text{ cm/min}]$ have a misfit value to our data of less than 0.111 ns, with the minimum at 0.131 cm/min. We think that this wide interval of possible $K_s$ is over-estimated by our rough error determination.

![Image of parameter uncertainties on the arrival time difference](image)

Figure 4. Influence of the parameter uncertainties on the arrival time difference, between the arrival time generated with each parameter and the arrival time generated with optimal parameters.

4. Constant Head Infiltration

4.1. Numerical Example

In the second case, a water layer of 5 cm above the ground is kept constant during the entire experiment. Similarly as above, using the same van Genuchten parameters as in the first synthetic example ($\theta_r=0.43$, $\theta_s=0.07$, $\alpha=0.019 \text{ cm}^{-1}$, $n=8.67$, and $K_s=0.120 \text{ cm/min}$), we modeled infiltration of water inside a ring infiltrometer by applying a constant pressure head of 5 cm to the top node during 10 minutes. The permittivity profiles are presented in Figure 5a, with each curve plotted every 10 s as in the previous case. Figure 5b shows the radargram simulated with GprMax2D. As can be seen, the reflection labeled A, describing the position of the infiltration front, is returning at increasing times because infiltration is being constantly fed by the constant ponding depth, contrary to the previous falling head case. In Figure 5c, we computed the TWT of the wetting front, using the ray path algorithm and the picking of A reflection coming from the radargram in Figure 5b.

Assuming the four van Genuchten parameters $\theta_r$, $\theta_s$, $\alpha$, and $n$ are known, we inverted for the saturated hydraulic conductivity by minimizing the differences between the arrival times of the wetting front reflection obtained by the ray path-algorithm and the arrival times picked from the radargram in Figure 5b. The objective function was minimized for $K_s=0.128 \text{ cm/min}$, which is again larger than the value used for simulating the data: $K_s=0.120 \text{ cm/min}$. 

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4.2. Experimental Example

The experiment took place in the same quarry of Fontainebleau sand as the previous experiment. The middle of antennae was positioned in the middle of the ring \((x = 30 \text{ cm in Fig. 1})\). The recorded GPR data are shown in Figure 6. We recorded during 80 minutes, with a time window of 30 ns (in Fig. 6, we only present a part of the radargram), transmitting and receiving each 5 seconds and with no stacking. In the radargram in Figure 6, we subtracted the average trace and applied an AGC gain to the data. We used the van Genuchten parameters determined in the laboratory using the hanging column experiments, and we measured on sand samples an initial volumetric water content of \(\theta_i = 0.07 \pm 0.02\).

![Figure 5](image)

Figure 5. Constant head infiltration with 5 cm of water. a) Permittivity profiles, each curve is plotted every 10 s. Crosses represent the inflection points. b) Radargram simulated with GprMax2D, reflection A is the wetting front, B is the direct wave, A' and A'' are multiples. c) Two Way Travel Time, computed with an optical path algorithm directly from the permittivity profiles.
Figure 6. GPR data acquired during a constant head (5 cm) infiltration. Reflection A is the reflection coming from the wetting front.

In the profile presented in Figure 6, the arrival time of reflection A ranges from 0 at the beginning of the experiment to about 6 ns after 10 min. We picked the arrival time of the reflection A peak and computed the objective function using the same procedure as described before. We obtained the minimum of the objective function for $K_s = 0.110$ cm/min. This value has to be again compared with the one obtained by the disk infiltrometer experiment, $K_{Disk} = 0.108 \pm 0.01$ cm/min. Using the same procedure as presented in the earlier field example, we found a total quadratic error of $\delta_{tot} = 0.131$ ns, which gives a range of possible values for the saturated hydraulic conductivity, $K_s = [0.035; 0.213]$ cm/min.

5. Toward a Three Parameter Inversion

From our uncertainty analysis (Fig. 4), we pointed out the high sensitivity of our inversion to the parameter $\alpha$. The solution may be in inverting our GPR measurements for $\alpha$ and $n$ in addition to $K_s$, assuming $\theta_s$ and $\theta_r$ known. We computed misfit diagrams of the arrival times, to analyze which parameters can be easily inverted by our algorithm and which cannot. The procedure of calculating misfit diagrams is presented in Figure 6d. We carried out this analysis for both infiltration cases. We set one of the three parameters and plotted the misfit diagram for the two others. In Figure 6, we present the misfit diagrams obtained for the constant head infiltration. In Figure 6, we present similar diagrams for the falling head infiltration. When Hydrus-1D did not converge for a given combination of parameters, we set the misfit equal to 10 (dark red areas in Figures 6abc). The misfit diagram ($K_s, \alpha$) in Figure 6b does not exhibit a correlation between those two parameters, and informs us about the possibility of inverting ($K_s, \alpha$) while knowing $n$. The two other diagrams show the difficulty of obtaining the correct value of $n$. Indeed, this parameter is directly related to the slope of the retention curve $\partial \theta / \partial h$, and thus also to $\partial \theta / \partial z$ at the infiltration front. This information is not present in the TWT of the inflection points used for the misfit computations.

Figure 7 presents misfit diagrams for the falling head infiltration, calculated using the same procedure as above. The upper boundary condition in Hydrus-1D was set to a variable pressure
head. Note again that each time when Hydrus-1D did not converge, the misfit was set equal to 10 (dark red zones in the graph). Graphs 7a and 7c are completely different than in the previous case. They imply that having information on the evolution of the water layer thickness from the experiment itself is providing additional information that can help resolve the uniqueness problem for the $n$ parameter. This case needs to be explored in the future in more detail.

Figure 7. Misfit diagrams for the constant head boundary condition: a) $n$-$\alpha$ diagram, b) $K_s$-$\alpha$ diagram, and c) $K_s$-$n$ diagram. Black stars represent minimum.

Figure 8. Misfit diagrams for the falling head boundary condition: a) $n$-$\alpha$ diagram, b) $K_s$-$\alpha$ diagram, and c) $K_s$-$n$ diagram. Black stars represent minimum.
References