Repeated falling-head method for in situ measurements of saturated hydraulic conductivity using a single cylinder

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A simple, repeated falling-head (RFH) method, which uses an inexpensive portable cylinder to quickly determine field-saturated hydraulic conductivity (\(K_{fs}\)), is presented. A cylinder of radius \(r\), inserted vertically from the soil surface to a depth \(d\), is used as a water supply tank, through which water infiltrates into the soil. The falling-head test is repeated without allowing the cylinder to be drained entirely to eliminate the effects of the initial soil moisture conditions. Either change in the water level \((H)\) in the water supply cylinder or strong linearity between the infiltration flux \((q)\) and \(H\) can be used to determine \(K_{fs}\) directly or by multiplying the slope with an empirical ring-installation scaling length, \(L_g\); thus, no soil-dependent variable is necessary. To obtain the \(L_g\) value for various conditions, we carried out numerical experiments using HYDRUS (2D/3D) with a recently implemented reservoir boundary condition for six different soil textures, ranging from coarse soil (e.g., sand) to fine soil (e.g., silty clay loam), and two different ranges of water levels, \(15 \leq H \leq 25\) cm and \(2 \leq H \leq 6\) cm. Numerical results showed that the \(L_g\) has a strong linear dependence on \(d\) and \(r\), where \(L_g\) increases as \(d\) and \(r\) increase. We propose a new linear model to determine \(L_g\) with optimized coefficients for each \(H\) range. Numerical results showed no significant differences in the estimated \(K_{fs}\) after the second repetition, confirming that repeating the falling head test twice is sufficient. The newly obtained \(L_g\) values were then applied to the experimental data obtained at multiple fields having different soil textures. The \(K_{fs}\) values determined using the RFH method agreed well with those measured using the field two-ponding depths steady-state method and/or the FH method in the lab. These results demonstrate the reliability of the proposed RFH method.

1. Introduction

Hydraulic properties must be accurately determined to properly manage soil and optimize irrigation strategies, particularly where water resources are scarce and land degradation due to water erosion and salinization is severe. The field-saturated hydraulic conductivity \((K_{fs})\) is one of the most important soil hydraulic properties used in many hydrological (e.g., separating infiltration and runoff) and agricultural (e.g., determining water requirements in irrigation systems) applications. While several methods have been developed, many still require some soil texture-dependent parameters or rather expensive devices. In addition, since water is a precious resource in arid regions, there is a limit on the amount of water that can be used in experiments. There is, therefore, a great need to develop a quick, simple, and reliable method using an inexpensive portable device that does not require much water to determine \(K_{fs}\) in situ. Various steady-state and transient methods have been developed over the years. Steady-state methods include, for example, using a single-cylinder pressure infiltrometer \((\text{Reynolds and Elrick, 1990; Elrick and Reynolds, 1992; Reynolds et al., 2002; Angulo-Jaramillo et al., 2016})\). Transient methods include one-dimensional falling-head infiltration methods \((\text{Philip, 1992})\), early-time falling-head infiltration methods \((\text{Elrick et al., 1995})\), single-ring infiltrometer methods \((\text{Wu and Pan, 1997; Wu et al., 1999})\), simplified falling-head infiltration methods \((\text{Bagarello et al., 2004; Bagarello and Sgroi, 2007})\).

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Bagrak infiltration methods (Lassabatère et al., 2006; Bagrak et al., 2017), bottomless bucket falling-head methods (Nimmo et al., 2009), or modified bottomless bucket methods (Mirus and Perkins, 2012), among others.

A method based on the steady-state infiltration rate, measured for a single water level (H) (the one-ponding depth approach) in a cylinder of radius r inserted vertically from the soil surface to a depth d, requires a soil texture-dependent parameter (α*), defined as Kfs/qm, where qm is the matrix flux potential (Elrick and Reynolds, 1992). In contrast, the two-ponding depths (TPD) approach (Nimmo and Elrick, 1990) does not require α* and measures steady-state infiltration fluxes, q1 and q2, at two water levels, H1 and H2, respectively, and obtains Kfs using

$$K_{fs} = G \left( \frac{q_2 - q_1}{H_2 - H_1} \right)$$

(1)

The dimensionless shape factor, G, accounts for complex interactions among r, d, H, the three-dimensional soil capillarity effect, and gravity. When G values are plotted as a function of d/r, they approximate a straight line with a slope (C1) and an intercept (C2) as follows:

$$G = C_1 \left( \frac{d}{r} \right) + C_2$$

(2)

Reynolds and Elrick (1990) determined that C1 = 0.316 and C2 = 0.184 (referred to as the RE coefficients throughout the manuscript) using the numerical experiments for the steady-state, constant-head, and single-cylinder infiltration tests performed with cylinders having r = 5, 7.5, and 10 cm, and d = 3 and 5 cm. These coefficients were obtained by averaging the G values for given H+r-d combinations for four different soil types. According to them, the G values estimated using Eq. (2) are applicable to 5 ≤ r ≤ 10 cm, 3 ≤ d ≤ 5 cm, and 5 ≤ H ≤ 25 cm. Although the RE coefficients were validated only for a narrow range of d/r for steady-state constant-head infiltration conditions, they have been widely used, even for d/r outside of this range and for both steady and transient methods (e.g., Youngs et al., 1995; Wu and Pan, 1997; Mirus and Perkins, 2012; Bagrak et al., 2019). Compared with transient falling head methods, steady-stage methods require longer measurement time to attain steady state conditions and an additional device, such as a Marriott flask, to keep the water level constant.

Among many transient methods, the simplified falling-head method (Bagrak et al., 2004) is a simple method that uses a single cylinder but requires soil texture-dependent parameter α*, in addition to values such as the difference between the saturated and initial water contents, Δθ. The Beerkat infiltration method (Lassabatère et al., 2006; Bagrak et al., 2017) is a simple, inexpensive method that uses small cups as cylinders through which water infiltrates into the soil to determine, for example, a complete hydraulic characterization such as the hydraulic conductivity curve based on the relation by Brooks and Corey (1964). However, the data analysis procedure requires the knowledge of soil texture-dependent particle size distribution, which makes it difficult for quick in-situ determination. The bottomless bucket falling-head method (Nimmo et al., 2009) is a simple method with a relatively simple data analysis procedure. The formula derived for the constant head test is applied to a very short duration of a falling head test where the rate of change in H equals the infiltration flux, q. A soil texture-dependent macroscopic capillary length (λ) (White and Suly, 1987) is still required in addition to a ring-installation scaling length, L0, that accounts for complex interactions among r, d, H, the three-dimensional soil capillarity effect, and gravity, similar to G used in the steady-state tests. Since Noborio et al. (2018) followed their fall-in-head test the data analysis of Nimmo et al. (2009), a value of L0 is still required, but instead of using soil texture-dependent parameters α* or λ, they used the slope between q and H to estimate Kfs. Because q was obtained by time differentiation of H that was approximated by a quadratic function of t, q became a linear function of time, and Kfs became time-dependent. This means that the final estimate of Kfs depends on the duration of the experiment. Kfs of the field soil may not be constant as soil structure can change with time due to many factors, such as swelling and shrinkage under wetting and drying. However, like many similar studies that developed in situ measurement methods, we developed the method assuming that soil is rigid and homogenous so that Kfs is constant and time-independent. Thus, it would be inconvenient to have time-dependent Kfs. Additionally, although Nimmo et al. (2009) and Noborio et al. (2018) used the RE coefficients to estimate L0 values, using the coefficients derived for steady state conditions may be questionable under transient conditions.

In this study, we propose a simple in-situ falling head test, a repeated falling head (RFH) test, in which the effect of initial soil wetness is eliminated by repeating the falling head multiple times, with a simple data analysis that does not require any soil texture-dependent parameters. The data analysis procedure requires the value of L0 following Nimmo et al. (2009) and Noborio et al. (2018) but eliminates some drawbacks of their analyses presented above. Details about the data analysis procedure are presented in Materials and Methods. We calibrated L0 under transient conditions for various initial water contents, ranges of H, combinations of d and r, and soil types through numerical experiments using the HYDRUS (2D/3D) software package with a recently implemented reservoir boundary condition (Siminçek et al., 2018). We also conducted field and laboratory experiments to compare Kfs obtained with RFH using the newly estimated L0 with the ones measured with the TPD and falling head methods for soil cores from six soils of different textures.

2. Materials and methods

2.1. Procedure of repeated falling head method

As a variant of available transient falling head field tests to determine Kfs, we propose, in this study, a repeated falling head (RFH) method, in which a cylinder of a radius r inserted vertically from the soil surface to a depth d is used. In RFH, the falling-head test is repeated without allowing the cylinder to be drained entirely to eliminate the effects of the initial soil moisture conditions. Detailed data analysis procedures that do not require soil texture-dependent parameters for RFH are given below. To determine the number of necessary repetitions in RFH, field experiments and numerical simulations were used.

2.2. Theory of repeated falling head method

Now, consider the situation of performing an in-situ falling head test, where a cylinder of a radius r is inserted from the surface to a depth d and the height of the water inside the column H [L]. As water infiltrates into the soil, H starts to drop. Assuming that the RE equation, originally developed for the constant head test, can be applied to the very short time period of the falling head test, the hydraulic conductivity can be estimated from the following equation, as given by Nimmo et al. (2009) and Noborio et al. (2018):

$$q = \left( \frac{dH}{dt} \right) = K_{fs} \frac{L_0 + \lambda + H}{L_0}$$

(3)

This equation accounts for additional factors besides gravity, such as sorption, horizontal spread of water below the column, and other factors that cause Kfs to systematically deviates from q. Note that by replacing L0 and λ with the column height and the bottom constant pressure head, respectively, this equation becomes equivalent to the Darcy equation used to analyze the laboratory falling head test. The following data analysis procedure of RFH relies on Eq. (3).

By integrating Eq. (3) from t = 0 to a given elapsed time t [T], during which the water level drops from the initial level of H0 [L] to H, we get the time evolution of H as follows:
If \( L_g \) is given, \( K_f \) can be directly obtained by fitting Eq. (4) to \( H-t \) data.

By taking a derivative of Eq. (3) with respect to \( H \), we can numerically obtain \( \frac{dq}{dH} \) from \( H \) to \( H \) instead of using the actual elapsed time. Instead of using parameter values proposed by Reynolds and Elrick (1990) for a constant head test to determine \( L_g \) based on the linear model given as Eq. (2), in this study, we propose a new linear model, which is more suitable for in-situ falling head tests, through numerical experiments.

In practice, after obtaining \( L_g \), there are mainly two approaches to determining \( K_f \) using data collected from RHF. One approach is, as mentioned above, to use Eq. (4) directly (Approach 1). Another approach is to determine \( K_f \) from Eq. (5), in which the value of \( dq/dH \) is required. If \( H \) is monitored at a constant or approximately constant time interval, instead of applying Eq. (4), we may take the central difference of \( H \) to obtain \( q_t \) for each \( H \). We can then obtain the slope \( dq/dH \) with simple linear regression. \( K_f \) is finally obtained from Eq. (5) with \( L_g \) (Approach 2). Mathematically, this approach should give almost the same result as the first approach, but does not require any curve fitting. One can initially interpolate \( H-t \) data with a polynomial function if data show large fluctuations or extremely uneven time intervals. Noborio et al. (2018), for example, used a quadratic function to interpolate their \( H-t \) data with the following equation:

\[
H = A_1 t^2 + A_2 t + A_3
\]

where \( A_1 [\text{LT}^{-2}] \), \( A_2 [\text{LT}^{-1}] \), and \( A_3 [\text{L}] \) are empirical fitting parameters. One of the drawbacks of this approach is that if \( dq/dH \) is derived analytically from Eq. (6), the final \( K_f \) becomes time-dependent (see Eq. (12) of Noborio et al. (2018)). \( K_f \) of the field soil may not be constant as soil structure may change with time due to many factors, such as swelling and shrinkage. However, in this study, like in many similar studies, we assume the soil is rigid so that \( K_f \) is not time-dependent. Thus it is convenient to have time-dependent \( K_f \). To eliminate the time dependency of \( K_f \), Noborio et al. (2018) suggested using half the time from start to end instead of using the actual elapsed time. Instead of using an analytical solution, we can numerically obtain \( q_t-H \) data from the interpolated \( H-t \) data simply by taking a slope, \( dH/dt \), for given \( H \). After developing a \( q_t-H \) relationship, simple linear regression can be used to obtain \( dq/dH \) (Approach 3).

### 2.3. Numerical experiments to obtain \( L_g \) values for various conditions

To determine \( K_f \) using either Eq. (4) or (5), we need the \( L_g \) value. In this study, the \( L_g \) values for various conditions were obtained through numerical simulations. The HYDRUS (2D/3D) software package with a recently implemented reservoir boundary condition (Simane et al., 2018), which allows us to mimic the falling head process, was used for the numerical experiments. We assume that the soil is homogeneous and isotropic, and is characterized by the default parameters (Carsel and Parrish, 1988) of the van Genuchten-Mualem soil hydraulic functions (van Genuchten, 1980) provided in the HYDRUS (2D/3D) software package for six soils ranging from fine to coarse textures: sand, loamy sand, sandy loam, loam, silt loam, and silty clay loam according to the USDA soil texture classification (Table 1). The van Genuchten-Mualem soil hydraulic functions are given as:

\[
S_e = \frac{\theta - \theta_s}{\theta_i - \theta_s} = (1 + \alpha h^n)^{-m}
\]

\[
K(h) = K_s S_e \left[ 1 - \left( \frac{1 - S_e^{1/m}}{m} \right)^n \right]^{2/n}
\]

Table 1

<table>
<thead>
<tr>
<th>Soil</th>
<th>( \theta_i )</th>
<th>( \theta_l )</th>
<th>( \alpha )</th>
<th>( n )</th>
<th>( L )</th>
<th>( K_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.045</td>
<td>0.43</td>
<td>0.145</td>
<td>2.68</td>
<td>0.5</td>
<td>0.495</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>0.057</td>
<td>0.41</td>
<td>0.124</td>
<td>2.28</td>
<td>0.5</td>
<td>0.2432</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.065</td>
<td>0.41</td>
<td>0.075</td>
<td>1.89</td>
<td>0.5</td>
<td>0.073</td>
</tr>
<tr>
<td>Loam</td>
<td>0.078</td>
<td>0.43</td>
<td>0.036</td>
<td>1.56</td>
<td>0.5</td>
<td>0.0173</td>
</tr>
<tr>
<td>Silt loam</td>
<td>0.067</td>
<td>0.45</td>
<td>0.02</td>
<td>1.41</td>
<td>0.5</td>
<td>0.0075</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.089</td>
<td>0.43</td>
<td>0.01</td>
<td>1.23</td>
<td>0.5</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

where \( \theta_i [\text{L}^3 \text{L}^{-3}] \) is the residual water content, \( \theta_s [\text{L}^3 \text{L}^{-3}] \) is the saturated water content, \( K_s [\text{L}^2 \text{T}^{-1}] \) is the saturated hydraulic conductivity, \( \alpha [\text{L}^{-1}] \) is a shape coefficient, \( m \), \( n \), and \( l \) are dimensionless empirical coefficients, \( h [\text{L}] \) is the soil water pressure head, \( \theta [\text{L}^3 \text{L}^{-3}] \) is the water content, and \( S_e \) is the effective saturation [-].

The axisymmetrical simulation domain had a radius of 50 cm and a depth of 60 cm. This domain size was selected to prevent the infiltration front from reaching the domain’s boundaries for all soil types considered in this study during the numerical experiments. The domain was discretized into finite elements using the HYDRUS mesh generator with a targeted element size of 0.5 cm. The bottom of the simulation domain was set as a free drainage boundary condition, along which the pressure head gradients were set to zero. No-flow boundary conditions were assigned to the sides of the domain and the soil surface, except inside of the cylinder (similar to Wu and Pan, 1997; Simane et al., 2018; Bagarello et al., 2019; Di Prima et al., 2019). The soil surface in the cylinder was set to a reservoir boundary condition (Simane et al., 2018), along which pressure is dynamically updated in response to infiltration. The pressure head at the soil surface corresponds to \( H \).

We carried out 168 numerical experiments for various combinations of \( H \), \( r \), \( d \), and soil types. Four \( r \) (2, 4, 7.5, 14 cm), four \( d \) (3, 4, 5, 7 cm), and six soils were used for a high and wide range of \( H (15 \leq H \leq 25 \text{ cm}) \) where \( H_0 = 25 \text{ cm} \) and \( H_1 = 15 \text{ cm} \): denoted 15H25 hereafter, resulting in a total of 96 cases. Three \( r \) (2, 7.5, 14 cm), four \( d \) (2, 3, 4, 5 cm), and six soils were used for a low and narrow range of \( H (2 < H \leq 6 \text{ cm}) \) where \( H_0 = 0 \text{ cm} \) and \( H_1 = 2 \text{ cm} \); denoted 2H6 hereafter), resulting in a total of 72 cases. Most of these values were those employed in previous studies (Reynolds and Elrick, 1990; Di Prima et al., 2018; Noborio et al., 2018; Shiraki et al., 2019) except for \( d = 2 \text{ cm} \), which was tested in our field experiment. The choice of the \( H \) range depends on the available cylinder and other factors. Considering it should be easily carried, the cylinder should be at most 30 cm long. We do not want to carry much water for measurement in arid regions where water is scarce. Thus, a low and narrow range is preferred. In this study, 15H26 is used as an example for the former, while 2H6 is used for the latter. It should be noted that because \( L_g \) depends on the used \( H \) range, one needs to perform numerical simulations before measurements if different \( H \) ranges are used so that appropriate \( L_g \) values can be determined.

The initial condition was set to either a uniform pressure head with a given \( h \) or a hydrostatic equilibrium, having a pressure head of –100 cm at the soil surface, depending on the purpose of each simulation. In the simulations, as demonstrated in Fig. 1a and Fig. 2a, the initial \( H \) at the reservoir boundary was set to a value slightly higher than the highest height of measurement, \( H_0 \) (i.e., 27 cm for 15H25 and 7 cm for 2H6). As the simulation proceeds, \( H \) at the reservoir boundary drops because of infiltration. When \( H \) reaches a lower preset value (i.e., 13 cm for 15H25 and 1 cm for 2H6), \( H \) is returned to the initial \( H \) by applying a large flux rate (i.e., adding water into a reservoir) at the boundary to repeat the
falling head process. Initially, the refill process was repeated four times in each case to investigate if the effect of the initial condition could be reduced as we repeated. The subsequent falling head periods were denoted as Cycles 1, 2, 3, and 4. The number of repetitions can be reduced if the results for successive repetitions are not significantly different. In the numerical experiments, because HYDRUS outputs both $q_s$ and $H$ at the same print time, the slope $\frac{dq_s}{dH}$ can be directly obtained from linear regression between $q_s$ and $H$. The $L_g$ value for each case was then obtained by solving Eq. (5) for $L_g$ using $K_s$ used in the simulations. A multi-variable linear model is finally used to determine $L_g$ using $d$ and $r$ as independent variables for each range of $H$.

The accuracy of fitted $L_g$ values in numerical experiments was evaluated using the root mean square error (RMSE) defined as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (L_g - L_{\text{reg}})^2}$$

(8)

where $N$ is the total number of simulations, and $L_{\text{reg}}$ is $L_g$ estimated using determined linear coefficients. Since the true $K_s$ value is known in the numerical experiments, the accuracy of $K_s$ estimated using $L_{\text{reg}}$ can be quantified using the relative error, $E_{\%}$, defined as

$$E_{\%}(\%) = 100 \left| 1 - \frac{K_s}{K_{\text{true}}} \right|$$

(9)

Fig. 1. The numerical solution of (a) the time evolution of $H$ and $q_s$, and (b) a relationship between $H$ and $q_s$ for a high and wide range of $H$ (15H25), $r = 7.5$ cm, $d = 3$ cm, and sandy loam. The hydrostatic equilibrium with a pressure head of -100 cm at the soil surface and dry condition (a constant $h$ of $-5000$ cm) were used as the initial condition in (b).

Fig. 2. The numerical solution of (a) the time evolution of $H$ and $q_s$, and (b) a relationship between $H$ and $q_s$ for a low and narrow range of $H$ (2H6), $r = 7.5$ cm, $d = 3$ cm, and silty clay loam. The hydrostatic equilibrium with a pressure head of -100 cm at the soil surface and dry condition (a constant $h$ of -5000 cm) were used as the initial condition in (b).
2.4. Field experiments

Field experiments are subject to various uncertainties and errors associated with, for example, readings, an inclination of cylinders, temperature fluctuations, swelling of the soil, entrapped air, macro-pores, gaps between cylinder and soil, and disturbance of the soil surface by poured water, which are difficult to replicate in numerical experiments. To evaluate the accuracy of RFH under actual field conditions, we carried out RFH tests at six experimental sites with various soil textures in Japan and compared the results with those using a common two-ponding depth (TPD) approach and a laboratory falling head (FH) method.

The six sites, as listed in Table 2, include a sandy field at the Arid Land Research Center, Tottori University (ALRC, N 35°32'06", E 134°12'42"), three Faculty of Agriculture field plots, Tottori University (FATU, N 35°30'54", E 134°10'13"), a field at Tokyo University of Agriculture and Technology (TUAT, N 35°41'02", E 139°29'10"), and a field at the Ikuta Campus of Meiji University (SAMU, N 35°36'38", E 139°32'52""). Table 2 summarizes the fractions of sand, silt, and clay, bulk density, and soil classification according to the USDA classification of soils at the experimental field sites. The r and d values used in the experiments are also listed in Table 2. Steel cylinders with r = 5.5 and 14 cm were used at the FATU site, while transparent acrylic cylinders with scales were used at the other sites. When the H drop lasted several days, cylinder tops were covered with a plastic sheet to prevent the effects of rain and evaporation. Gaps between the soil and the cylinder formed during cylinder insertion were filled with in-situ soil using sticks. Soil surface disturbance that occurred during water application was minimized by placing a paper towel on the soil surface inside the cylinder, which was later removed during the experiment. Cylinders were covered with a heat insulation jacket with high reflectance to avoid significant temperature effects when the measurement took a long time. Because the H drop rate was fast at the ALRC and FATU-1 sites where the soil texture was sand, changes in H were recorded by video images. Core samples with a diameter of 5.1 cm and a height of 5 cm (resulting in 100 cm³ in volume) were taken at the ALRC and three FATU sites below the cylinders after the RFH tests were concluded to determine the Kₑ values using a falling head method in the laboratory.

The TPD tests were carried out at the ALRC, FATU-1, and FATU-2 sites. In the TPD tests, a constant float water was used in the water supply container to mitigate the effects of temperature and atmospheric pressure fluctuations. Although experiments were continued until an almost constant flux was achieved, there is no standard protocol for judging if a steady-state condition is reached. The time evolution of q₁ was then fitted with:

\[ q₁ = B₁ \exp(B₂t) + q_a \]  

where B₁ [LT⁻¹] and B₂ [T⁻¹] are fitting parameters to estimate the steady state qₑ [LT⁻¹] for each of two water levels, Hᵢ (i = 1, and 2), used in Eq. (1). The TPD tests were carried out after the RFH tests using the same cylinders without removing/reinstalling them. Since the TPD tests were performed in this study purely for comparison, we do not intend to discuss the validity of continuing measurements for long periods over several days.

3. Results and discussions

3.1. Numerical experiments to estimate Lₑ

Fig. 1 and 2 shows, as examples, H(t), q₁(t), and qₑ(H) computed for sandy loam, 15H25, r = 7.5 cm, and d = 3 cm, and, for silty clay loam, 2H6, r = 7.5 cm, and d = 3 cm, respectively. Fig. 1b and Fig. 2b indicate that, regardless of the soil type and H range, repeated cycles give almost the same or similar qₑ(H) except for Cycle 1. Fig. 1b and Fig. 2b also shows qₑ(H) for two different initial conditions: a hydrostatic equilibrium with a pressure head of ~100 cm at the soil surface and a constant h of ~500 cm. Fig. 3 shows the slope, dqₑ/dH, for six different soil texture types, 15H25, r = 7.5 cm, and Cycles 2, 3, and 4, as a function of d. As can be seen, dqₑ/dH values among different cycles are almost the same regardless of the soil types and d. Same results were obtained for 2H6. These results confirm that, except for Cycle 1, the initial condition’s effect may be diminished and that repeating the falling-head experiment twice is sufficient for RFH. By terminating the number of repetitions at two, one can save considerable time and water compared with TPD while minimizing the effect of the initial water content. The data from Cycle 2 are then used further to obtain Lₑ. Fig. 4 depicts the Lₑ values for both 15H25 and 2H6 as a function of d for 168 numerical experiments (96 for 15H25 and 72 for 2H6) under different combinations of radii and soil textures. It shows that Lₑ depends linearly on d for all H ranges regardless of the soil texture. It also shows that Lₑ increases as r increases, indicating that Lₑ is a function of both d and r. As Lₑ values depend on the used H range, Lₑ was regressed using the following linear model for each H range:

\[ Lₑ = D₁d + D₂r + D₃ \]  

There are three fitting parameters required in this model, while there are only two parameters fitted to estimate G using the RE model (Eq. (2)). Fitted D₁ [-], D₂ [-], and D₃ [L⁻¹] were 0.774, 0.294, and 0.663, respectively, for 2H6, and those for 15H25 were 0.926, 0.427, and 0.446, respectively. Fitted lines for each r are also depicted in Fig. 4, demonstrating that the proposed linear model (Eq. (11)) can be used to predict Lₑ accurately. Fig. 4 also shows predicted Lₑ (grey lines) for each H range using the RE coefficients for D₁ (=0.993) and D₂ (=0.578) with D₃ = 0, indicating that the RE coefficients overestimate Kₑ under RFH. The reliability of the linear model was indicated by small errors where RMSE and Eₑ for 2H6 were 0.155 and 2.61%, respectively, and 0.176 and 1.78% for 15H25, respectively. Note that these coefficients, unlike \( α \), were valid regardless of the soil type.

3.2. Field experiments

All the Kₑ and Kᵢ values determined experimentally are, in the remainder of the paper, adjusted to those at 20 °C (e.g., Hopmans and Dane, 1986) to eliminate the effect of temperature following Klute (1965), in which the temperature dependence of water viscosity is used. Some measurements took days to reach a steady state for the constant head field infiltration experiments. In such cases, we adjusted Kₑ using an average temperature of the measurement duration. For the lab experiments, the temperature of drained water was used to adjust the Kᵢ values.
3.2.1. Effect of number of cycles

As an example, Fig. 5a depicts the time evolution of $H$ observed for 2H6, $r = 5.5$ cm, and $d = 3$ cm at the FATU-2 site. In Fig. 5b, $H$ is plotted against time, $t' [T]$, elapsed from the time $t_0 [T]$ when the water level reached $H = 6$ cm during the falling head in each cycle, i.e., $H(t')$ curves were almost identical except for Cycle 1, which also supports our conclusion of using Cycle 2 to determine $K_{fs}$ from the above numerical experiments. A curve drawn in Fig. 5b is Eq. (4) fitted to data from Cycle 2, demonstrating that Eq. (4) can accurately describe $H(t')$. Using the $L_g$ value computed with the coefficients obtained for 2H6, $K_{fs}$ was estimated as 0.0247 cm min$^{-1}$. At FATU-2, it took about 120 min to complete two cycles, and the volume of water used was at least 1.3 L. Based on our numerical experiments and some field experiments, we only repeated the falling head process twice in RFH for the rest of the experiments presented in this manuscript.

3.2.2. Comparison with the laboratory falling head and field Two-Ponding depths methods

The TPD method was performed right after the RFH measurement using the same cylinder without removing it at some experimental sites.
Fig. 5. The \( H \) for a combination of 2H6, \( r = 5.5 \) cm, \( d = 3 \) cm, and FATU-2 (Table 2) (a) as a function of elapsed time \( t \) and (b) as a function of elapsed time \( t' \) since the start of each \( H = 6 \) cm falling head cycle (b).

Table 3 summarizes the experimental conditions of TPD performed at our experimental sites with the resulting \( K_s \). At ALRC and FATU-1 sites, the TPD tests were performed twice with different water levels. For example, at the FATU-1 site, the TPD method was performed with constant water levels \( H_1 = 16.6 \) cm and \( H_2 = 21.5 \) cm for conditions with a radius \( r = 5.1 \) cm and an insertion depth \( d = 4 \) cm. Temporal changes in the infiltration flux \( q_s \) for \( H_1 \) and \( H_2 \) are plotted in Fig. 6. By fitting Eq. (10), \( q_{s1} = 0.875 \text{ cm min}^{-1} \) for \( H_1 \) and \( q_{s2} = 1.075 \text{ cm min}^{-1} \) for \( H_2 \) were determined with the RMSE for both less than 0.006 cm min \(^{-1} \). Using the RE coefficients \( C_1 = 0.316, C_2 = 0.184 \), we obtained \( G = 0.432 \) using Eq. (2) for the TPD method. The \( K_{s1} \) value determined using Eq. (1) with \( r = 5.1 \) cm was then 0.198 cm min \(^{-1} \), while \( K_{s2} \) determined by the RFH at the FATU-1 site was 0.181 cm min \(^{-1} \). At this site, after completing both RFH and TPD tests, core sampling was performed at a depth of 5 cm in the cylinder to determine the laboratory \( K_s \), which resulted in 0.163 cm min \(^{-1} \). The relative deviation between \( K_{s1} \) determined by RFH and TPD was less than 15%. It has to be noted that the total time required for the TPD method was over 500 min (Fig. 6) because we had to perform the infiltration tests twice for two water levels, which was 25 times longer than the 20 min required by the RFH method at the FATU-1 site. To save time, the test could be stopped when \( dq_s/dt \) is, for example, less than half of the initial slope and estimate the asymptote using Eq. (10), but results may be less reliable.

Figs. 7 and 8 plot \( K_s \) obtained using the data of Cycle 2 of the RFH experiment based on the proposed data analysis for 15H25 and 2H6, respectively, and those obtained using the standard laboratory FH and field TPD methods for all experimental sites tested in this study. For 15H25, only Approach 1 was used for RFH, while for 2H6, all three approaches were used. For all soils, from coarse to fine, most points fall closely along a 1:1 line depicted as a solid line in Figs. 7 and 8, with a maximum deviation of less than 25%, which is considered “accurate” based on Reynolds (2013), suggesting that the proposed method is applicable to a variety of soil types. The average deviation from \( K_s \) obtained using the FH method was 10.8%, while the average discrepancy from the TPD-measured \( K_s \) was 11.4%. Those deviations are acceptable, given that \( K_s \) can vary by a few orders of magnitude, even for samples of the same soil texture. Fig. 8 shows that all three approaches result in almost the same results suggesting that any approach can be taken. The overall results indicate that the RFH method with newly determined \( L_s \) is as reliable as the standard laboratory FH and field TPD methods. Given the simpler experimental setup and analytical method, we suggest using the RFH method in case \( K_s \) needs to be determined in situ.

3.2.3. Choice of \( H \) range

The RFH method was performed at the TUAT and SAMU sites with two different \( H \) ranges, respectively. The test conditions are as follows:
At the TUAT site, the obtained \( K_{fs} \) were (1) 0.102 and (2) 0.112 cm min\(^{-1}\), respectively, while at the SAMU site, they were (1) 0.799 and (2) 0.823 cm min\(^{-1}\), respectively. For both sites, relative differences between the two conditions are less than 9%. In both cases, the water used was less than 2.5 L, and the measurement time was shorter by about one-third for 2H6 compared to 15H25. For sandy soils, 15H25 (or a similar \( H \) range) is recommended because of the rapid \( H \) change, while a low and narrow range, such as 2H6, should be adopted for clayey soils to shorten measurement time.

4. Conclusions

In this study, we propose a repeated falling head method to determine field saturated hydraulic conductivity, \( K_{fs} \). A ring-installation scaling length, \( L_g \), required in the data analysis procedure was calibrated with numerical experiments using the HYDRUS (2D/3D) program. The following conclusions are drawn from this study by comparing the RFH method with the standard laboratory falling head (FH) and field two-point depth (TPD) methods for various experimental conditions and soil types.

1. Strong linearity between the water level, \( H \), in the water supply cylinder and the infiltration flux \( q_s \) during the second cycle of the RFH method, regardless of the initial soil wetness, allows us to adopt the data analysis requiring \( L_g \) proposed by Nimmo et al. (2009) and Noborio et al. (2018). The effect of the initial soil wetness can be eliminated after two repetitions by keeping the cylinder from being
completely drained. Thus, repeating the falling head experiment twice is sufficient in the RFH method.

2. HYDRUS (2D/3D) with a newly implemented reservoir boundary condition allows one to simulate the RFH method. Because the saturated hydraulic conductivity is known in the simulations, the $L_d$ values can be obtained for various conditions for the RFH method. Numerical experiments show that $L_d$ has a strong linear dependence on $d$ and $r$, where $L_d$ increases as they increase and is not soil-texture dependent. Multivariate linear regression analyses provided optimized coefficients for each $H$ range for a linear model proposed in this study. One should employ different linear coefficients for different $H$ ranges.

3. The $K_s$ values determined for various soil types with the newly determined $L_d$ for two $H$ ranges in the RFH method were comparable to those obtained from the laboratory FH and field TPD methods, confirming the reliability of the proposed method. The maximum deviation between the proposed and standard methods’ results was much less than 25%, which can be considered “accurate.”

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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