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HORIZONTAL INFILTRATION REVISITED USING PARAMETER ESTIMATION

Jirka Šimůnek[1], Jan W. Hopmans[2], D. R. Nielsen[2], and M. Th. van Genuchten[1]

A parameter estimation approach that combines a numerical code for unsaturated flow with a nonlinear optimization method was used to analyze horizontal infiltration data for Columbia silt loam and Hesperia sandy loam. The data were previously investigated by Nielsen et al. (1962) using the analytical method of Bruce and Klute (1956). As with the original analysis, water content profiles at the different times could be analyzed accurately in a simultaneous fashion only when the applied pressure head at the column boundary was close to saturation (−0.02 m). For much lower boundary pressures (−0.50 and −1.00 m), water content profiles for the different times had to be analyzed independently. Excellent agreement was obtained between diffusivities calculated from the same water content profiles using either the analytical method of Bruce and Klute or the numerical parameter estimation technique. However, in addition to diffusivities, the numerical parameter estimation analysis also provided estimates of the entire soil-water retention and hydraulic conductivity functions. Numerical analysis of the experimental data produced functional forms of the hydraulic properties, compared with point values, when using the analytical analysis. (Soil Science 2000;165:708–717)

Key words: Horizontal infiltration, parameter optimization, Bruce and Klute method, soil hydraulic properties.

INFILTRATION of water into an initially dry, horizontal soil column forms the basis of a popular laboratory method for determining the soil water diffusivity function (Bruce and Klute, 1956; Nielsen et al., 1962; Jackson, 1963; Vachaud, 1967; Whisler et al., 1968; Smiles et al., 1980; Clothier et al., 1983; Klute and Dirksen, 1986). The method, first introduced by Bruce and Klute (1956), involves infiltration (absorption) of water into a horizontal column of soil followed by destructive gravimetric sampling to obtain the spatial water content distribution at a fixed time. Subsequent modifications by Vachaud (1967) and Whisler et al. (1968) require the water content to be measured as a function of time at one or more fixed positions using a nondestructive method for continuously monitoring the water content using, for example, gamma attenuation.

The method of Bruce and Klute (1956), referred to here as the BK method, is based on the Boltzman transform of the diffusivity form of the unsaturated flow equation. For horizontal flow, the governing equation is given by

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left(D(\theta)\frac{\partial\theta}{\partial x}\right) \tag{1}$$

where θ is the water content (L^3L^{-3}), D is diffusivity (L^2T^{-1}), x is the spatial coordinate (L), and t is time (T). Equation (1) implies that Darcy's law is valid for unsaturated flow, whereas we also assume that a unique relationship exists between the water content and the pressure head (Nielsen et al., 1962). The initial and boundary conditions for horizontal infiltration are as follows:

$$\begin{aligned} \theta(x,t) &= \theta_i \qquad x > 0, t = 0 \\ \theta(x,t) &= \theta_0 \qquad x = 0, t > 0 \\ \theta(x,t) &= \theta_i \qquad x = \infty, t > 0 \end{aligned} \tag{2}$$

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[1] George E. Brown, Jr. Salinity Laboratory, 450 W. Big Springs Road, USDA-ARS, Riverside, CA 92507. Dr. Šimůnek is corresponding author. E-mail: jsimunek@ussl.ars. usda.gov.

[2] Hydrology Program, University of California, Davis, CA.

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in which the initial and boundary water contents are θ_i and θ_0, respectively, assumed constant with $\theta_i < \theta_0$. Using the Boltzmann transformation:

$$\lambda(\theta) = xt^{-1/2} \tag{3}$$

The partial differential Eq. (1) is transformed into the ordinary differential equation:

$$-\frac{\lambda}{2}\frac{d\theta}{d\lambda}=\frac{d}{d\lambda}\left(D\frac{d\theta}{d\lambda}\right). \tag{4}$$

Integration of Eq. (4) and using the initial and boundary conditions of Eq. (2) yields the following equation for the soil-water diffusivity:

$$D(\theta)=-\frac{1}{2}\left(\frac{d\lambda}{d\theta}\bigg|_{\theta}\right)\int_{\theta_i}^{\theta}\lambda d\theta \tag{5}$$

or, in terms of x at some point in time, t:

$$D(\theta) = -\frac{1}{2t}\left(\frac{dx}{d\theta}\bigg|_{\theta}\right)\int_{\theta_i}^{\theta}x d\theta \tag{6}$$

From the water content distribution, $\theta(x)$, either using Eq. (6) directly or transformed into a $\lambda(\theta)$ profile, one can calculate the soil-water diffusivity as a function of the water content. This method, or minor modifications thereof, has been used in many studies, as cited earlier, to analyze horizontal infiltration data, as well as for evaporation experiments (Arya et al., 1975; van Grinsven et al., 1985), to yield $D(\theta)$.

The BK method was quite popular in the 1960s, 1970s, and even in the early 1980s because the procedure requires very few computations. During this period of time, computers were still largely unavailable or were used infrequently as tools for data analysis of soil hydraulic measurements. Data such as those needed for the BK method can now be analyzed in a much more convenient and accurate manner using numerical inversion or parameter estimation (Kool et al., 1987; Hopmans et al., 2001). Parameter estimation techniques have recently been used for analysis of a variety of laboratory and field experiments, including one- and multi-step outflow experiments, evaporation methods, upward infiltration methods, and instantaneous profile experiments, as well as tension disc and cone penetrometer infiltration experiments. See Hopmans et al., (2001) for specific references on these particular methods. Moreover, although analyses based on Eq. (5) accomplish the diffusivity-water content function, numerical inversion can provide additional information about the water retention curve and the hydraulic conductivity function as well.

In this paper we will use the parameter estimation approach to analyze horizontal infiltration data previously presented and analyzed using the BK method (Nielsen et al., 1962). Results obtained by numerical inversion will be compared with results obtained using the original BK analysis. Furthermore, we will use this technique to infer the two complementary hydraulic relationships.

MATERIALS AND METHODS

Experimental

Details about the experimental technique are given in Nielsen et al. (1962). Here we summarize only the experimental information that is relevant to our analysis. We used data for horizontal infiltration into a Columbia silt loam and a Hesperia sandy loam. The air-dried soils, packed to the same average bulk density in 3.2-cm-diameter cylinders composed of 1-cm sections, were subjected to horizontal infiltration for different time periods so that the wetting front penetrated to various distances. Infiltration of water into the soils was controlled by imposing a negative pressure head at the boundary through a fritted glass bead plate. The pressure was held constant during each run. At the desired time, the water supply was discontinued, and the water content of each 1-cm section was determined gravimetrically. Gravimetric water contents were converted to volumetric values using the average bulk density of the entire column.

Parameter Estimation

Water flow in the numerical scheme is described using a mixed formulation of the Richards' equation

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K(h)\frac{\partial h}{\partial x}\right) \tag{7}$$

where h is the pressure head (L) and K is the hydraulic conductivity (LT^{-1}). The soil hydraulic properties are described with the van Genuchten-Mualem model (van Genuchten, 1980):

$$S_e(h) = \frac{\theta(h)-\theta_r}{\theta_s-\theta_r} = \frac{1}{(1+|\alpha h|^n)^m} \tag{8}$$

$$K(\theta) = K_s S_e^l [1-(1-S_e^{1/m})^m]^2 \tag{9}$$

where S_e is effective fluid saturation (−), K_s is the saturated hydraulic conductivity (LT^{-1}), θ_r and θ_s denote the residual and saturated water contents (L^3L^{-3}), respectively, l is the pore-connectivity parameter (−), and α (L^{-1}), n (−), and m ($=1 - 1/n$) (−) are empirical shape parameters. The hydraulic functions contain six unknown parameters: θ_r, θ_s, α, n, l and K_s. The soil water diffusivity function is formally defined by:

$$D(\theta) = \frac{K(\theta)}{C(\theta)} = \frac{K(\theta)}{d\theta/dh} \qquad (10)$$

where C is the hydraulic capacity (L^{-1}), being the slope of the soil water characteristic curve $\theta(h)$.

Equation (7), subject to the initial (constant water content) and boundary (constant pressure head) conditions, was solved numerically using the HYDRUS-1D code (Šimůnek et al., 1998). This code also has parameter estimation capabilities. Minimization of the merit or objective function that measures the agreement between measured and modeled water contents was accomplished within HYDRUS-1D by using the Levenberg-Marquardt, nonlinear minimization method (Marquardt 1963). The merit function was taken as a weighted least-squares estimator. The weighting coefficients v were given by

$$v = \frac{1}{n\,\sigma^2} \qquad (11)$$

where n is the number of water content data points and σ^2 is the variance of measured water contents. The above approach views the merit function as the average weighted squared deviation normalized by measurement variances σ^2.

RESULTS AND DISCUSSIONS

Optimization of Water Content Profiles

We analyzed the measured water content profiles, $\theta(x)$, for Columbia silt loam obtained for supply pressure heads, h_0, of -2 cm at three different times of 88, 344, and 740 minutes, and for $h_0 = -100$ at times of 441, 4182, and 28224 minutes. For Hesperia sandy loam, the times were 158, 620, and 1467 minutes for $h_0 = -2$ cm and 4820 and 23677 minutes for $h_0 = -50$ cm. This gave 11 different optimization scenarios. For each soil and each supply pressure head, we also optimized the water content profiles at the different times simultaneously. The five optimized soil hydraulic parameters (the residual water content θ_r was fixed to be equal to zero) obtained for the different cases are presented in Table 1. The optimization runs were always restarted with different initial guesses of the optimized parameters to ensure, as much as possible, reaching the global minimum. Table 1 presents parameters for runs having the lowest values of the objective func-

TABLE 1

Parameter estimation results for all analyzed experiments

Soil	Boundary pressure (cm)	Time (min)	θ_s	α (cm)	n	K_s (cm/min)	l	Merit Function
Columbia	−2	All	0.462	0.0309	2.12	0.162	4.03	0.152
		88	0.445	0.0208	1.87	0.176	14.1	0.0026
		344	0.455	0.0261	2.06	0.153	6.47	0.0074
		740	0.463	0.0296	2.15	0.157	4.45	0.0120
Columbia	−100	All	0.676	0.0290	1.61	0.0527	−0.808	0.230
		441	0.605	0.0219	1.69	0.846	3.56	0.0060
		4,182	0.554	0.0204	1.61	0.673	4.38	0.0054
		28,224	0.772	0.0342	1.66	1.10	2.34	0.0196
		All	0.457*	0.0640	1.20	0.917	0.458	0.295
		441	0.457*	0.0123	1.64	0.0555	3.61	0.0053
		4,182	0.457*	0.0162	1.43	0.0626	2.42	0.0091
		28,224	0.457*	0.0197	1.34	0.0277	0.582	0.0133
Hesperia	−2	All	0.394	0.0325	1.54	0.114	1.77	0.0728
		158	0.384	0.276	1.44	0.118	2.48	0.0305
		620	0.389	0.0308	1.53	0.114	2.01	0.0043
		1,467	0.396	0.0307	1.54	0.106	1.76	0.0171
Hesperia	−50	All	0.502	0.0327	1.81	0.0455	1.02	0.0984
		4,820	0.512	0.0291	2.10	0.265	3.62	0.0163
		23,677	0.484	0.0353	1.68	0.0681	1.57	0.0435
		All	0.390*	0.0295	1.43	0.0180	0.071	0.0997
		4,820	0.390*	0.0206	1.83	0.0205	2.63	0.0173
		23,677	0.390*	0.0193	1.96	0.0144	2.72	0.0444

*Fixed

tion. Nevertheless, we have no complete guarantee that the global minimum of the objective function was obtained in each case. The measurement variances, σ^2, of measured water content profiles for Columbia silt loam obtained for supply pressure heads, h_0, of -2 and -100 cm and for Hesperia sandy loam for -2 and -50 cm were calculated as follows: 0.003896, 0.00242, 0.00392, and 0.00209, respectively.

Measured and optimized water content profiles for Columbia silt loam wet at $h_0 = -2$ cm are shown in Fig. 1. Even with all three profiles being optimized simultaneously, the correspondence between measured and fitted water content profiles was excellent. Our results are almost identical to those obtained by Nielsen et al. (1962) using the Boltzmann transformation (Eq. (3)). Optimizations were less successful for Columbia silt loam wet at $h_0 = -100$ cm (Fig. 2). None of the water content profiles was fitted well when all three distributions were optimized simultaneously (results not shown; see much larger merit function value in Table 1). Each profile had to be optimized separately in order to obtain close correspondence between the measured and fitted values. The soil hydraulic parameters optimized using data for the two early times (441 and 4182 min) resulted in significant overprediction of the moisture front for the third time (28224 min) (dashed and dashed-dotted lines in Fig. 2). Optimizing the soil hydraulic parameters using data at t=28224 minutes similarly resulted in a too slow advance of the moisture front at the two earlier times. Again, these results correspond closely with those obtained by Nielsen et al. (1962). Nielsen et al. (1962) also discussed possible reasons why Eq. (1) was unable to predict water profiles for greater negative pressures for all three times simultaneously. For example, Eq. (1) assumes isothermal conditions, whereas soil heat is evolved when water wets soil. Eq. (1) also assumes a single-phase flow whereas soil water movement should be recognized as a 2-phase fluid problem (Nielsen et al., 1962). Nielsen et al. (1962) also state that "the assumption involving the validity of Darcy's law for unsaturated transient flow problems remains open to question."

Very similar results were obtained for Hesperia sandy loam. All three water content profiles could again be well optimized simultaneously when the soil was wet at -2 cm (Fig. 3), but each profile had to be optimized independently when the soil was wet at -50 cm (Fig. 4). The soil hydraulic parameters optimized using early time data (4820 min) again caused an overprediction of the moisture front at the later time (23677 min), whereas the soil hydraulic parameters optimized for t=23677 minutes produced an advance of the wetting front at t=4820 minutes that was too slow

For the above analysis, we used all water content profiles as presented by Nielsen et al. (1962), i.e., those obtained for a supply pressure head close to saturation (-2 cm) as well as those resulting from relatively low supply pressure heads (-50 or -100 cm). The diffusion-analog of soil water flow (Eq. (1)) requires that the $\theta(x)$ profiles collapse to a unique $\theta(\lambda)$. If not, then the experiments have, for a variety of reasons (several of which were discussed by Nielsen et al., 1962), failed to obey a diffusion-like process that is inherent in the mathematical description of Eqs. (1) and (7). From the analyzed water content profiles,

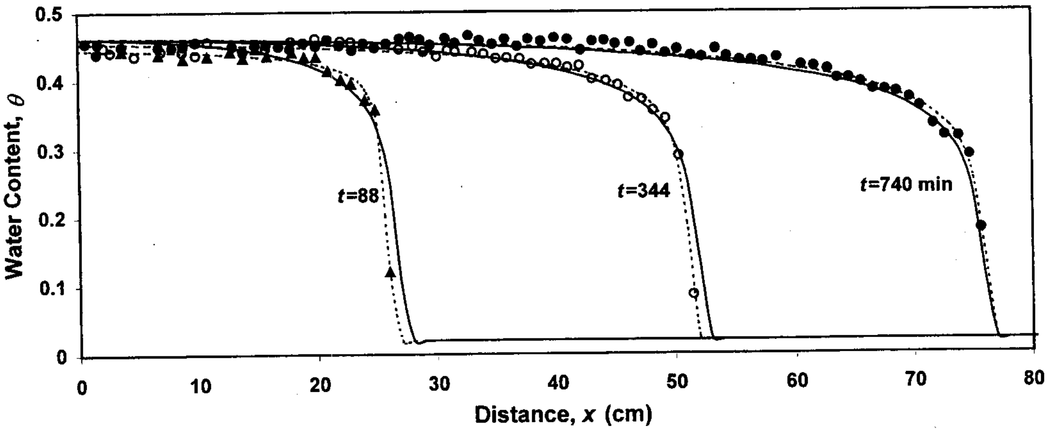

Fig. 1. Observed and fitted water content profiles for Columbia silt loam wet at a supply pressure head, h_0, of -2 cm. Solid and dashed lines were optimized against three observed water content profiles simultaneously and independently, respectively.

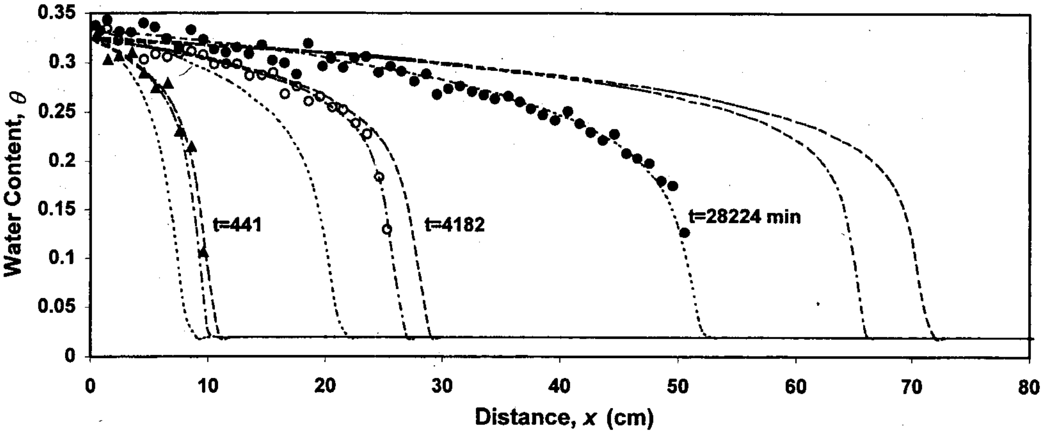

Fig. 2. Observed and fitted water content profiles for Columbia silt loam wet at a supply pressure head, h_0, of −100 cm. The dashed, dashed-dotted, and dotted lines were calculated from parameters optimized using the first (441 min), second (4152 min) and third (28224 min) profile, respectively.

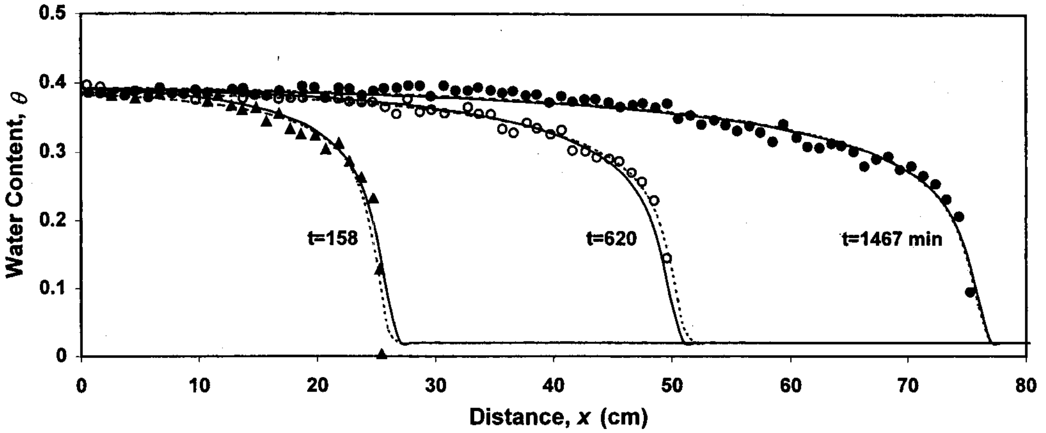

Fig. 3. Observed and fitted water content profiles for Hesperia sandy loam wet at a supply pressure head, h_0, of −2 cm. Solid and dashed lines were optimized against three observed water content profiles simultaneously and independently, respectively.

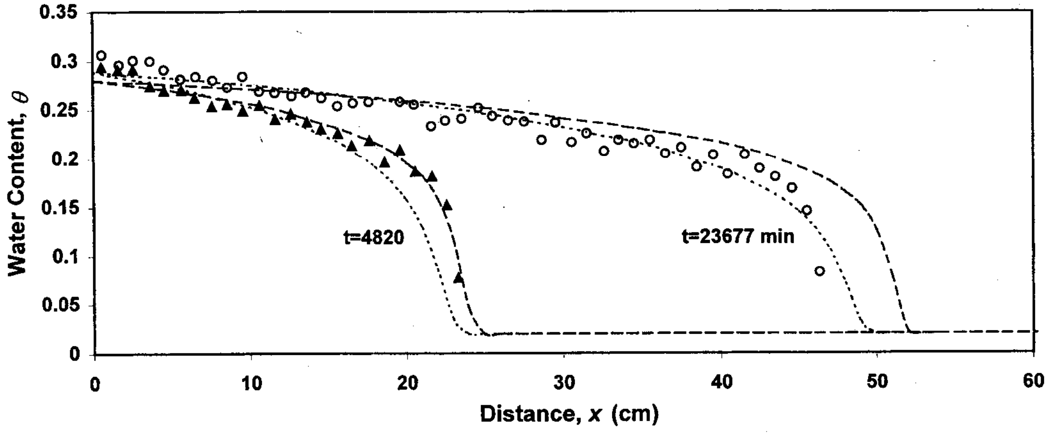

Fig. 4. Observed and fitted water content profiles for Hesperia sandy loam wet at a supply pressure head, h_0, of −50 cm. The dashed and dotted lines were calculated from parameters optimized using the first (4820 min) and second (23677 min) profile, respectively.

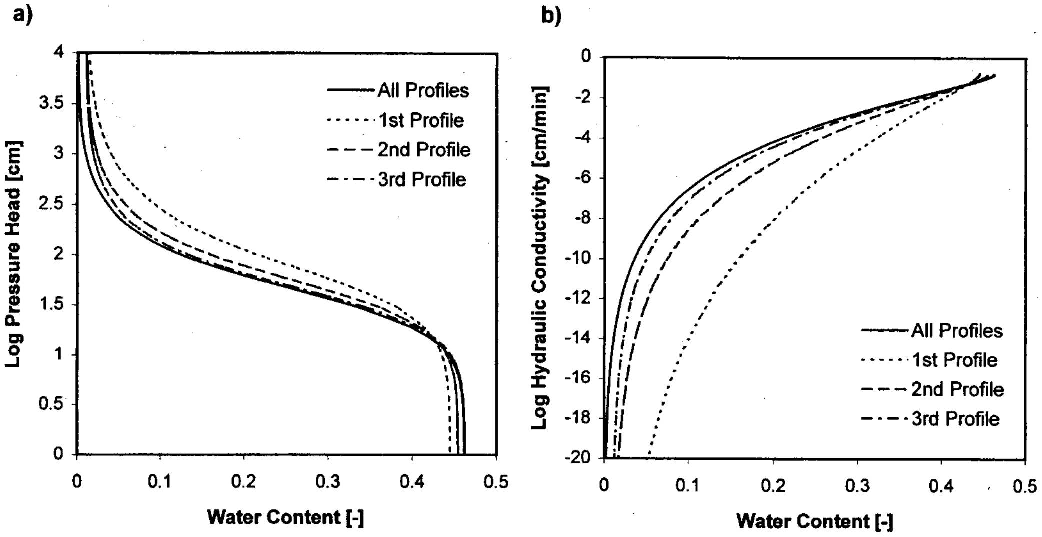

Fig. 7. Retention (a) and hydraulic conductivity (b) functions optimized for Columbia silt loam wet at −2 cm using all water content profiles simultaneously and each profile independently.

ductivity with decreasing water content (Fig. 7b). Not enough data points are present at the earliest time to characterize the water content front properly and, hence, to optimize the shape of the hydraulic conductivity function accurately.

Similar results were obtained within the measurement range (i.e., for $\theta < 0.34$) for the functions calculated from profiles wet at −100 cm (Fig. 8). However, results beyond the measurement range (i.e, for $\theta > 0.34$) are clearly suspect. Saturated water contents, and corresponding saturated hydraulic conductivities, were significantly overestimated in all optimizations for experiments wet at $h_0 = -100$ (see also Table 1). These two variables cannot be estimated clearly from infiltration data initiated using the boundary pressures far from saturation. Although the parameters had to be optimized separately for each time profile, the soil hydraulic functions remained in a very narrow range within the measurement range.

Even better results (in terms of the very narrow range of the soil hydraulic functions for the different optimizations) were obtained for Hesperia sandy loam (Figs. 9 and 10). Again, saturated

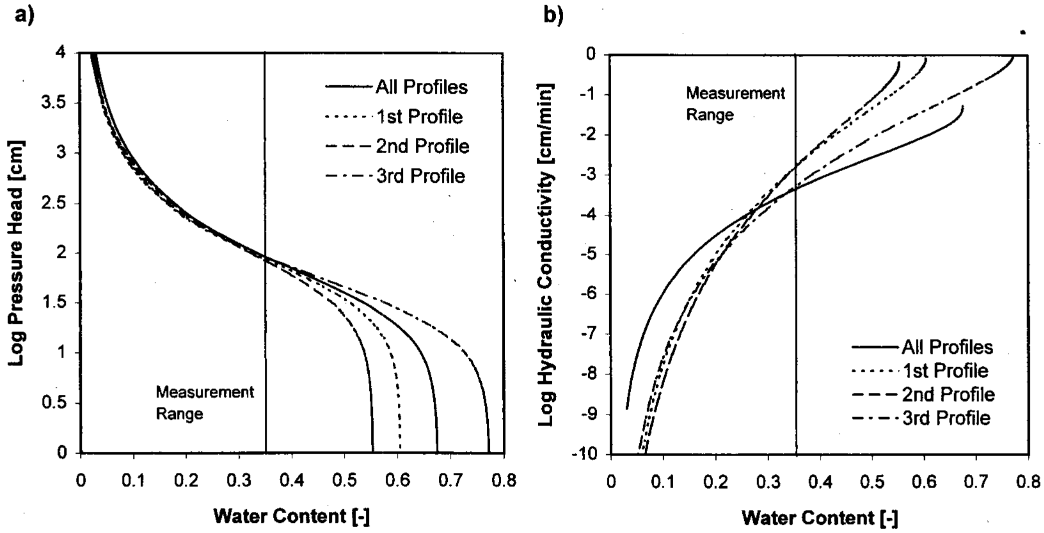

Fig. 8. Retention (a) and hydraulic conductivity (b) functions optimized for Columbia silt loam wet at −100 cm using all water content profiles simultaneously and each profile independently.

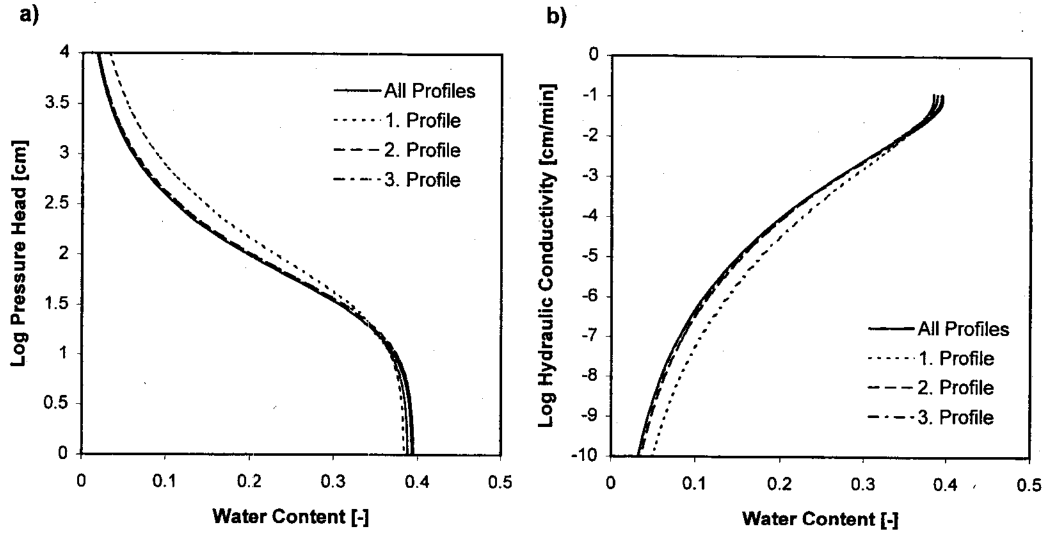

Fig. 9. Retention (a) and hydraulic conductivity (b) functions optimized for Hesperia sandy loam wet at −2 cm using all water content profiles simultaneously and each profile independently.

water contents and saturated hydraulic conductivities cannot be estimated reliably from infiltration data initiated using the boundary pressures far from saturation.

CONCLUSION

Similar to the BK analysis of Nielsen et al. (1962), water content profiles at the different times could be optimized accurately in a simultaneous fashion only when the applied pressure head was close to saturation (−2 cm). For much lower boundary pressures (−50 and −100 cm), water content profiles for the different times had to be optimized independently. Simultaneous optimization of two (for Hesperia sandy loam) or three (for Columbia silt loam) profiles in those cases resulted in poor agreement between measured and fitted water contents. From the point of view of parameter optimization, experiments with higher (close to saturation) supply pressure

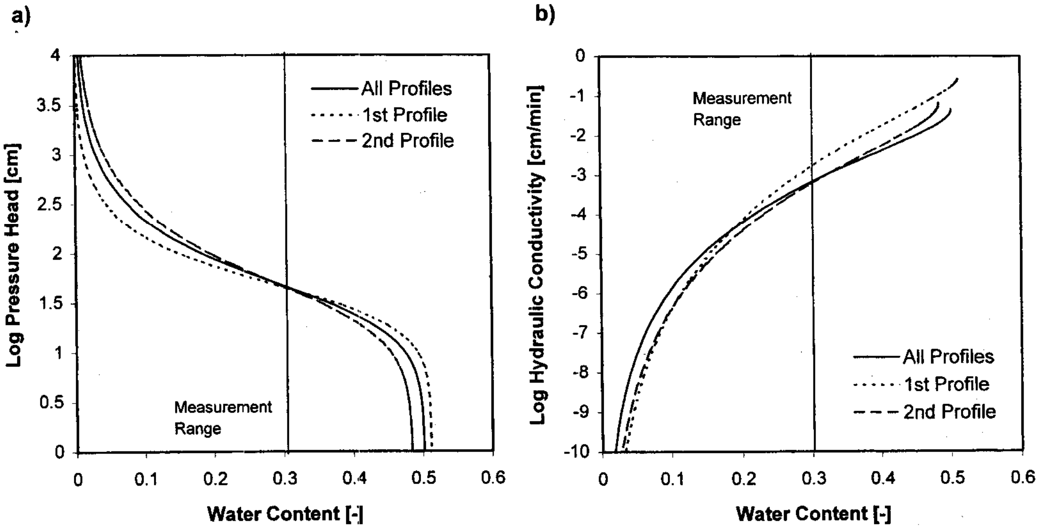

Fig. 10. Retention (a) and hydraulic conductivity (b) functions optimized for Hesperia sandy loam wet at −50 cm using all water content profiles simultaneously and each profile independently.

heads are preferable since (i) they cover a wider range of water contents and (ii) they seem to adhere better to the Boltzmann transform (Eq. (3)), at least for the data used here.

Diffusivities obtained analytically and numerically from the same water content profiles corresponded excellently. An additional advantage of the numerical analysis was that it resulted in functional forms of diffusivities and extended the water content range of predicted diffusivities. In addition to diffusivities, the numerical parameter estimation analysis of the horizontal infiltration experiments provided estimates of all soil hydraulic parameters defining the soil water retention and hydraulic conductivity functions. An additional advantage of the method is that when weights in the merit function correspond with the measurement errors, the numerical analysis provides confidence intervals for the optimized parameters. Also, numerical inversion, contrary to the analytical analysis, does not limit experimentalists to homogeneous initial conditions and time-invariant boundary supply pressures.

This paper focuses on the numerical analysis of horizontal infiltration data. Numerical inversion can be extended easily to other experiments. For example, horizontal infiltration experiments are often combined with solute transport (e.g., Smiles et al., 1978; Bond, 1986). Available numerical programs such as HYDRUS-1D (Šimůnek et al., 1998) have the capability to analyze transient water flow and solute transport experiments either sequentially or simultaneously and could be used immediately to analyze these types of experiments Šimůnek et al., 2001). Vachaud (1968), using gamma attenuation to measure water contents at fixed positions, allowed water to redistribute in the soil column after initial horizontal infiltration. Such data set can be used immediately to estimate soil hydraulic parameters for hysteretic soils (Šimůnek and van Genuchten, 1999).

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