A Mathematical View of Water Table Fluctuations in a Shallow Aquifer in Brazil
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Abstract
Detailed monitoring of the groundwater table can provide important data about both short- and long-term aquifer processes, including information useful for estimating recharge and facilitating groundwater modeling and remediation efforts. In this paper, we present results of 4 years (2002 to 2005) of monitoring groundwater water levels in the Rio Claro Aquifer using observation wells drilled at the Rio Claro campus of São Paulo State University in Brazil. The data were used to follow natural periodic fluctuations in the water table, specifically those resulting from earth tides and seasonal recharge cycles. Statistical analyses included methods of time-series analysis using Fourier analysis, cross-correlation, and R/S analysis. Relationships could be established between rainfall and well recovery, as well as the persistence and degree of autocorrelation of the water table variations. We further used numerical solutions of the Richards equation to obtain estimates of the recharge rate and seasonable groundwater fluctuations. Seasonable soil moisture transit times through the vadose zone obtained with the numerical solution were very close to those obtained with the cross-correlation analysis. We also employed a little-used deep drainage boundary condition to obtain estimates of seasonable water table fluctuations, which were found to be consistent with observed transient groundwater levels during the period of study.

Introduction
Groundwater monitoring constitutes an important tool for aquifer management by providing information about the current state and trends of various hydrological processes, as well as recording responses to both anthropogenic activities and natural phenomena. Detailed monitoring further provides insight into the nature of groundwater table fluctuations in response to such processes as precipitation, evapotranspiration and recharge, regional flow and pumping, barometric pressure changes, earth tides, and even earthquakes (Todd 1959; Bredehoeft 1967; Roeloffs 1988; Healy and Cook 2002; Zeumann et al. 2009; Cutillo and Bredehoeft 2011; Chen et al. 2013; and numerous references therein). Understanding the processes that cause variations in the water table and the ability to monitor groundwater levels is critical for estimating recharge, whether on the scale of an aquifer to assess groundwater resources and management in general, or for local situations such as for assessing groundwater contamination.

This paper presents the results of detailed monitoring of the groundwater table from 2002 to 2005 in wells of the Rio Claro Aquifer, drilled on the campus of São Paulo State University (UNESP) in the city of Rio Claro in the Brazilian state of São Paulo. The data were analyzed statistically using a combination of public-domain software and software specifically developed for this study. We used methods of time-series analysis (Fourier analysis, cross-correlation, and R/S analysis) to verify the periodicity of natural cycles such as those stemming from seasonal precipitation and earth tides, and to estimate relationships between rainfall and well recharge. The persistence and degree of autocorrelation of the water table variations were also evaluated. We further used one-dimensional (1D) vadose zone numerical flow modeling with a special deep drainage boundary condition to obtain estimates of the recharge rate, soil moisture transit times in the unsaturated zone, and resultant seasonable water table fluctuations. Results show that detailed monitoring provides valuable information contributing to a more comprehensive understanding of subsurface flow processes. The information may further facilitate remediation efforts for situations where vertical transport of contaminants, especially volatile constituents, is enhanced by frequent variations in the groundwater table (e.g., Neeper 2001; Stafford and Rixey 2011; Iglauer and Muggeridge 2013).
Groundwater levels were monitored in wells of the Rio Claro Aquifer, drilled for this purpose on the campus of São Paulo State University in the city of Rio Claro, Brazil. The Rio Claro Aquifer is a relatively shallow unconfined aquifer with groundwater tables being quite variable, generally between depths of about 6 and 10 m at the study site (Oliva et al. 2005). The aquifer consists mostly of Cenozoic sedimentary rocks of fluvial origin in the Rio Claro Formation (DAEE 1981). Sediments are composed of fine- to medium-grained sands with an abundant clay matrix, with the thickness of the aquifer varying from a few meters to up to 50 m. Recharge occurs in most of the aquifer, while discharge areas are located near the contact of the formation with Permian clay-rich sediments of the Corumbatá Formation of the Paraná Basin, as well as in narrow valleys.

Two wells located 80 m apart were drilled to penetrate and pass through the Rio Claro Aquifer for detailed monitoring of the groundwater table. The first well (IGCE-3) was drilled to a depth of 17 m at the end of March, 2001. The well was lined with polyvinyl chloride (PVC) casing, with a section of the screen/mesh extending from a depth of 5.6 m to the bottom of the well. The water table at the time of drilling was 7.4 m below the soil surface. This first well was monitored from April 2001 to April 2004, at which time the monitoring devices were transferred to another well (IGCE-6). That second well was drilled in September 2003 down to a depth of 20.1 m to the aquitard of the Corumbatá Formation, with the screen extending from a depth of 7.1 m to the bottom of the well. The water table in this second well at the time of drilling was 9.8 m below the soil surface.

Water Table Fluctuations and Pluviometric Data

Groundwater levels were monitored using a water column probe connected to a computer recording the water level every 5 s. Automated measurements were obtained using a pressure transducer having a range of 5 psi, equivalent to an approximately 3.5 m column of water. The transducer was vented to allow barometric corrections and installed below the water table in the well. The accuracy of the probe was 0.1% of its full range. Electrical signals from the probe were converted into digital values using a metering device equipped with a standard RS-232 interface for computer data transfer. The data were registered using a microcomputer running Wellplex, a software package capable of monitoring up to eight simultaneous devices, developed by the Basin Studies Laboratory (LEBAC) at UNESP University in Rio Claro.

Pluviometric data were obtained from daily total rainfall records maintained by the meteorological station of the Center for Environmental Studies and Planning (CEAPLA) at UNESP, located approximately 500 m from the observation wells. Figure 1 shows a plot of the precipitation data, as well as of measured water table data after time-series preprocessing to allow for corrections and filtering. Much of the preprocessing was done using the interactive Tsoft software package of Van Camp and Vauterin (2005). This software was found to be very useful for pre-treatment of our data, including for isolating unusual factors such as the presence of outliers, and for identifying very short temporal variations such as the effects of earthquakes on groundwater levels.

One difficulty during data acquisition was the presence of time-series gaps caused by temporary instrument failure. However, throughout 4 years of monitoring, the total data loss was less than 20%, which were filled by linear interpolation. This seems acceptable given the fact that the maximum interval of data loss was less than 3 d out of a total of 1460 d (4 years).

A second problem requiring correction involved the abrupt offset in values resulting from probe relocation from the first to the second well. This relocation was necessary to maintain the probe submerged and within the specified metering range. The shifts in position were corrected by adding (or subtracting) a constant value to (or from) all points in the time series after the first displaced value. This value was easily calculated from the difference between the value of the last point prior to offset and the measurement immediately afterwards. After making these corrections, we adopted an arbitrary reference datum situated at 633 m above mean sea level (approximately 9.5 m below the soil surface).

The original 5-s sampling interval resulted in a sequence of more than 20 million data points. These points were filtered to reduce the data to generate smaller files that were easier to manage. A new sample interval of 24 h was subsequently adopted to match the interval of...
available pluviometric data. The 5-s sampling interval was initially selected with the intent to record high-frequency phenomena such as those resulting from earthquakes (Todd 1959; Roeloffs 1988; Zeumann et al. 2009; Chen et al. 2013). We indeed detected several earthquakes with our monitoring system, including some from as far away as Alaska (results not further shown here).

**Observed Seasonal Variations and Earth Tides**

Figure 1 shows the major cycles identified during 4 years of monitoring the groundwater table (from January 2002 to December 2005). Since monitoring involved two different wells covering different periods of time, data were concatenated into a single curve. Seasonal variations in the water table are clearly a result of variations in annual rainfall, with the maximum levels increasing from 2002 to 2004, but decreasing in 2005. The response of the groundwater table is a result of recharge, which in turn is a function not only of the amount of rainfall but also of its distribution in time. For example, the longer duration of the summer rainy season (December to March) in 2003/2004 led to more recharge and a higher peak in the water table, whereas in early 2005 some of the rains were heavier but lasted less time, leading to less recharge.

The effects of earth tides on the water table data are shown in Figure 2. Although the effects of tides are more obvious on the water levels of oceans and other surface water bodies when compared with those of subsurface systems, forces generated by gravitational fields do cause deformation of rocks, leading to earth tides of the type shown in Figure 2. The effect of earth tides on water levels in wells in both unconfined and especially confined aquifers has been well documented (Todd 1959; Bredehoeft 1967; Roeloffs 1988; Rojstaczer and Agnew 1989; Neeper 2001; Zeumann et al. 2009, among others). According to Bredehoeft (1967), observations of fluctuations in water levels caused by earth tides have existed since the end of the 19th century, while Todd (1959) cited the work of Robinson (1939) describing variations in the water level of wells in Iowa and New Mexico.

The effects of earth tides on the groundwater table were clearly visible in our data (Figure 2), which is not surprising given the sensitivity of the instruments we used. Two cycles occurred each day, with the greatest amplitude occurring at approximately 3 or 4 am, and the smallest in the early morning at approximately 3 or 4 am. As noted by Todd (1959), the effect of these tides during new and full moons is quite regular, with the amplitude of the variations being greater than those during the first and last quarters when the cycles are less regular due to opposing gravitational forces resulting from the positions of the sun and moon relative to Earth.

**Analysis of Time Series**

The water table data were analyzed statistically using several methods of time-series analysis: Fourier analysis, cross-correlation, and R/S analysis. The methods and results are detailed in the following sections.

**Fourier Analysis**

A Fourier analysis may be used to reveal the frequencies of periodic and non-periodic functions, with the frequencies of the periodic functions determined by the coefficients in the Fourier series. Fourier analyses make it possible to analyze the components of the time series of the water table in the frequency mode, which in terms of our study should confirm the presence of two different types of periodic variations: one due to the season of the year involving annual recharge and drawdown of the aquifer, and one caused by earth tides producing two daily fluctuations. As our study involved data sampled at discrete time intervals, a discrete Fourier transform (Bracewell 1986; Fleming et al. 2002; Si 2007) was used:

\[
F(n) = \sum_{t=0}^{N-1} f(t) e^{-i\omega_0 t} \quad (1)
\]

where \(\omega_0 = 2\pi n/N\) for \(n = 0, 1, \ldots, N - 1\), resulting in a discrete set of Fourier frequencies. When this set of frequencies is plotted, the resulting graph is a periodogram or spectrogram, which identifies the contribution of the frequency of each of the periodic phenomena to the formation of the original signal.

For analysis of the seasonal water table variations, we used the filtered and corrected data time series. Calculation of the spectrum of frequencies of the periodic process using the discrete Fourier transform generated the spectrogram shown in Figure 3, portrayed on a log scale to enhance the spectral peaks. The horizontal coordinate is plotted in terms of cycles per day. Since seasonal variations occur only once a year, their frequency is 1/365 cycles/d, or approximately 0.0027 cycles/d. Figure 3 shows that the peak of the spectrogram of the entire time series indeed occurred at or very close to this value.

For analysis of the earth tides, only a portion of 2003 was used, corresponding to the peak of the seasonal cycle.
when the effect of tide was clearer. The groundwater table at that time was very stable and did not reflect much influence from either recharge or drawdown. The spectrogram is shown in Figure 4. The larger peak of frequency for a periodicity of 2 cycles/d and the smaller peak for a single cycle per day reflect the result of the twice-daily tide (K2) and the daily tide (K1), associated with the revolution of the Earth–Moon system around the center of mass and the rotation of the Earth around its own axis, respectively (Pugh 1987).

Cross-Correlation Analysis

Another useful method for analyzing temporal series in hydrogeological studies is cross-correlation quantifying the dependency of successive values over a period of time. Padilla and Pulido-Bosch (1995) and Larocque et al. (1998) used this method in their studies of the hydrodynamic characteristics of karst aquifers, including both spatial and temporal variations, as well as the relation of the hydrographs of rivers and creeks to discharge zones, estimation of specific storage and porosity, and the delay between input and output signals (such as rainfall and piezometric levels).

Consider two discrete time series, with one series, \( x_t \) \( (x_1, x_2, \ldots, x_n) \), representing the input to the system being investigated (the cause), and a second series, \( y_t \) \( (y_1, y_2, \ldots, y_n) \), the output (the result). The cross-correlation function is then given by the expressions (Padilla and Pulido-Bosch 1995):

\[
 r_{+k} = r_{xy} (k) = \frac{C_{xy} (k)}{\sqrt{C_x (0) C_y (0)}} \tag{2}
\]

\[
 r_{-k} = r_{yx} (k) = \frac{C_{yx} (k)}{\sqrt{C_x (0) C_y (0)}} \tag{3}
\]

where

\[
 C_{xy} (k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x}) (y_{t+k} - \bar{y}) \tag{4}
\]

\[
 C_{yx} (k) = \frac{1}{n} \sum_{t=1}^{n-k} (y_t - \bar{y}) (x_{t+k} - \bar{x}) \tag{5}
\]

\[
 C_x (0) = \frac{1}{n} \sum_{t=1}^{n} (x_t - \bar{x})^2 \tag{6}
\]

\[
 C_y (0) = \frac{1}{n} \sum_{t=1}^{n} (y_t - \bar{y})^2 \tag{7}
\]

in which \( \bar{x} \) and \( \bar{y} \) are the means of the \( x_t \) and \( y_t \) series, respectively, \( k = 0, 1, 2, \ldots, m \), with \( k \) being the time lag from one element to the next one, and where \( m \) is the cutting point that determines the interval over which the cross-correlation analysis is being carried out. The cutting point is generally chosen such that it includes the process whose verification is sought (Larocque et al. 1998). We note that correlation does not always imply cause as the correlation measures only the degree of the shared increase or decrease of two variables. However, both series \( x_t \) and \( y_t \) may be varying as a function of a third variable. For our study, we assumed that \( x_t \) is the cause of \( y_t \) due to previous knowledge of how the system works.

The cross-correlation function generates a graph (the correlogram), which shows the correlation (positive or negative) between the input and output elements for each time lag \( k \). The delay between the input and output series is defined as the time interval between \( k = 0 \) and the maximum value of the correlogram. The shorter this interval, the more rapidly the output series \( y_t \) reacts to a signal of the input series \( x_t \). If there is no time lag between the input and output, the influence of the former on the latter is instantaneous, and the maximum correlation is found for \( k = 0 \). In this study, the correlogram was calculated using the time series of rainfall data as input and the water table data as output (Figure 1). A cutting point value of 15.552.000 s (180 d) was chosen as only one major recharge peak occurs each year, a time window large enough to cover the recharge process. The two time series were used to calculate the mean time duration...
$\Delta t$ for the rainfall events to move through the vadose zone to the water table. The highest correlation value of 0.278 was obtained for a time period $\Delta t$ of 7,516,800 s, or 87 d (Figure 5). This is hence an estimate of the average time for rain water to percolate from the soil surface to the phreatic surface. We will later test this delay between rainfall and water table response by carrying out 1D transient simulations of water flow through the vadose zone.

**Rescaled Range (R/S) Analysis**

Another method known as rescale range analysis, or R/S analysis (Hurst et al. 1965), has been widely used to analyze time series of natural phenomena, including soil, hydrologic and meteorological processes (e.g., Mandelbrot and Wallis 1969; Christoforletti 1997; Breslin and Belwards 1999; Miranda and Andrade 2001; Peters et al. 2002; Favaretto 2004; Green et al. 2008). The analysis generates a value known as the Hurst exponent ($H$), which is able to distinguish between random and non-random time series, as well as provides information about the degree of autocorrelation of the series being considered. The R/S analysis is best described within the context of modeling a reservoir, for which it was originally created. If one considers that the input of water into a reservoir in year $u$ is represented by $\xi(u)$, the deviation from the mean $\xi(u) - \langle \xi \rangle_t$, for a specific year $t$ is calculated on the basis of the average over several years as follows:

$$\langle \xi \rangle_t = \frac{1}{t} \sum_{u=1}^{t} \xi(u)$$

Equation 8

The sum of the deviations, $X$, of the discharge from the mean is thus given by:

$$X(t, \tau) = \sum_{u=1}^{t} \{ \xi (u) - \langle \xi \rangle_t \}$$

Equation 9

The difference between the maximum and minimum values of $X$ is the range $R$, which for a reservoir is the difference between the maximum and minimum levels of water stored over a period of $\tau$ years. This range is expressed as:

$$R(\tau) = \max X(t, \tau) - \min X(t, \tau), \ (1 \leq t \leq \tau)$$

Equation 10

while the standard deviation, $S$, is given by

$$S = \sqrt{\frac{1}{N-1} \sum_{t=1}^{n} \{ \xi(t) - \langle \xi \rangle_t \}^2}$$

Equation 11

Dividing $R$ by $S$ for various intervals of the time-series data and calculating the mean (or expected value, $E$), Hurst et al. (1965) observed that the values converged to an exponential relationship:

$$E \left[ \frac{R(N)}{S(N)} \right] = kN^H \quad \text{with} \quad N \rightarrow \infty$$

Equation 12

where $k$ is a constant, $N$ is the number of observations, and $H$ is the Hurst exponent.

If $H = 0.5$, deviations of the values in the time series from the mean are random and hence independent of those of previous and subsequent elements. If $0.5 < H < 1$, the series is persistent, in which case deviations tend to remain stable over time (e.g., 1 year of flooding tends to be followed by another year of flooding, or a dry year is followed by another dry year). The probability of a value in the series deviating in the same direction (either positive or negative) as the previous one increases as $H$ increases. On the other hand, if $0 < H < 0.5$, deviations will be anti-persistent in that a positive deviation then tends to be followed by a negative deviation, and vice versa. The probability of this inversion of the deviation from the mean to occur increases as the value of $H$ decreases.

Breslin and Belwards (1999) developed an autocorrelation average, $C$, related to the Hurst exponent, using the equation:

$$C = 2^{2H-1} - 1$$

Equation 13

If $H = 0.5$, then $C = 0$, which implies that for a random series no autocorrelation exists between the measurements, and succeeding values are independent of each other. If $H > 0.5$ or $H < 0.5$, however, the correlation is either positive or negative, meaning that the system “remembers” previous events, and hence that past events will affect future events. The value of a given measurement is then affected by previous measurements, and the current measurement will also affect future results.

The $H$ exponent can be estimated by first establishing the size of the largest and smallest intervals of the time series being considered (the various $N$ values in Equation 12). Next the value of R/S is calculated for the largest interval (a value equal or close to that of the complete series). This series is then divided into two

![Cross-Correlation Value](Image)

Figure 5. Correlogram resulting from the analysis of water table and rainfall data.
Table 1
Results for Estimating the Hurst Exponent of the Measured Water Levels

<table>
<thead>
<tr>
<th>Number of Readings</th>
<th>Mean (RS)</th>
<th>Log 2 of Number of Readings</th>
<th>Log 2 of Mean RS</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3.491</td>
<td>3</td>
<td>1.804</td>
</tr>
<tr>
<td>16</td>
<td>6.941</td>
<td>4</td>
<td>2.795</td>
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<td>13.861</td>
<td>5</td>
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<td>27.708</td>
<td>6</td>
<td>4.792</td>
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<td>128</td>
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<td>7</td>
<td>5.793</td>
</tr>
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<td>256</td>
<td>110.458</td>
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<td>6.787</td>
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<tr>
<td>512</td>
<td>218.893</td>
<td>9</td>
<td>7.774</td>
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<tr>
<td>1024</td>
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<td>4096</td>
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<td>3533.701</td>
<td>13</td>
<td>11.787</td>
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<td>16,384</td>
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<td>32,768</td>
<td>10118.010</td>
<td>15</td>
<td>13.305</td>
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Table 2
Results for Estimating the Hurst Exponent of the Rainfall Data

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<th>Log 2 of Number of Readings</th>
<th>Log 2 of Mean RS</th>
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</thead>
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<tr>
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<td>11.026</td>
</tr>
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</table>

Figure 6. Graph of points in Tables 1 and 2 (with marks), together with the fitted lines, obtained with the R/S analysis for the water table data (a) and rainfall data (b).

and consistent with results obtained for other rainfall time series (e.g., Favaretto 2004; Peters and Christensen 2006).

The $H$ exponent for the water table data was found to be larger than the value for rainfall. This can be explained by the fact that the aquifer functions as a reservoir which, to a certain extent, integrates the rainfall variations over time and space. Therefore, for the various time scales, the effect of occasional dry periods interspersed by periods of rainfall are somewhat buffered by the aquifer as the infiltrating rain water is stored and released only slowly, and then also only in part to the aquifer. This leads to a greater autocorrelation of the water table values and parts, and the $R/S$ value of each of them is calculated, thus determining the mean. Each of these halves is subsequently halved again, with the process of calculating the mean of the two halves being repeated until the value of the original smallest interval is reached.

Converting both sides of Equation 12 to logarithms gives

$$\log \left( \frac{R(N)}{S(N)} \right) = \log k N^H = H \log N + \log k \quad (14)$$

Equation 14 shows that a log–log graph of the calculated values, or a linear graph of the logarithms of the values, will make it possible to estimate the value of $H$ from the slope of the straight line through simple linear regression.

The aforementioned procedure for the $H$ exponent was coded in a computer program and applied to both the water table data and the rainfall measurements. For the groundwater table, we used the same time series as before, but with temporal spacings of 1 h between readings. The shortest interval involved 8 readings and the longest interval 32,768 readings, the latter number being the power of 2 closest to the total number of readings (slightly more than 34,000). The points obtained in this manner are presented in Table 1. A graph of these points, along with the fitted line, is shown in Figure 6. The value of the Hurst exponent, which corresponds to the slope of the straight line, was 0.971. As this value is greater than 0.5, it shows that variations in the groundwater table are persistent in that the system has a memory of past levels, with strongly autocorrelated values. In comparison, the value of $H$ obtained by Hurst from an analysis of minimum water levels of the Nile River from the year 622 to 1469 was 0.91 (Mandelbrot and Wallis 1969).

Using the same process for the rainfall data produced the results in Table 2 and Figure 6. The $H$ exponent in this case was 0.766, again indicating a persistent phenomenon and consistent with results obtained for other rainfall time series (e.g., Favaretto 2004; Peters and Christensen 2006).

The $H$ exponent for the water table data was found to be larger than the value for rainfall. This can be explained by the fact that the aquifer functions as a reservoir which, to a certain extent, integrates the rainfall variations over time and space. Therefore, for the various time scales, the effect of occasional dry periods interspersed by periods of rainfall are somewhat buffered by the aquifer as the infiltrating rain water is stored and released only slowly, and then also only in part to the aquifer. This leads to a greater autocorrelation of the water table values and
suggests that the aquifer system has a relatively long “memory” of the water table fluctuations, thereby showing the “Joseph effect” of the series. This term, taken from the biblical story of the Prophet Joseph foreseeing 7 years of plenty followed by 7 years of famine in Egypt, was used by Mandelbrot and Wallis (1968) to describe the effect of persistence in hydrological time series with $H > 0.5$, in which an increase or decrease in values tends to follows the same direction as previous values.

Recharge Calculations

Further insight into recharge and the response of the water table was obtained by simulating infiltration and variably saturated flow in the soil profile during the 4-year experiment. Simulations were carried out using version 4.08 of the HYDRUS-1D software package (http://www.pc-progress.com/en/Default.aspx?hydrus-1d) of Šimůnek et al. (2008). This code has been used in several recent studies to estimate recharge processes (e.g., Lu et al. 2011; Assefa and Woodbury 2013). The calculations assumed applicability of the Richards equation to 1D vertical variably saturated flow subject to root water uptake by the grass cover at the site. We refer to the HYDRUS technical manual (Šimůnek et al. 2008) for a detailed description of the governing equations, as well as of various initial and boundary conditions that can be implemented.

For the recharge calculations, we assumed a vertical profile of 10 m containing an observation node at a depth of 9.5 m, being the approximate location of the pressure transducer. Atmospheric boundary conditions were assigned to the soil surface assuming runoff to occur when surface ponding exceeded 2 cm. Daily values of precipitation and potential evapotranspiration were used, in combination with the water stress response model of Feddes et al. (1978) to account for root water uptake by the grass cover. Daily potential evapotranspiration rates were calculated using the approach of Hargreaves (Hargreaves 1975; Jensen et al. 1997) based on available temperature data, the latitude (22.24 $^\circ$S) and altitude (643 m) of the site, the leaf area index of the grass cover (estimated to be 2.0) and the rooting depth (70 cm). Default parameter values of the water stress response functions were used based on literature values for grass (Feddes et al. 1978) as implemented in HYDRUS-1D. A linear decreasing root distribution from the soil surface (maximum) to a depth of 70 cm was assumed.

The van Genuchten–Mualem equations (van Genuchten 1980) were used to describe the unsaturated soil hydraulic properties. Soil hydraulic parameters (Table 3) were estimated from available soil texture data using the pedotransfer functions of Schaap et al. (1998) as implemented in HYDRUS-1D, but with measured values of the saturated hydraulic conductivity, $K_s$ (Alfaro Soto et al. 2007).

The initial condition was taken to be simply $-200$ cm in the upper part of the profile and, in a second simulation, an equilibrium distribution in the lower part of the profile with the water table at 800 cm. Only an approximate estimate of the initial condition was needed as we started the numerical calculations using local weather data on January 1, 2001 (taken to be $t = 0$ here), 1 year before the 4-year water table study period covering the period January 1, 2002 through December 31, 2005. This ensures that the adopted initial moisture profile at $t = 365$ d was as consistent as possible with the local soil and hydrologic conditions.

Several alternatives were considered for the lower boundary condition. Figure 7 shows the calculated recharge rate for a free-draining profile that neglects the presence of a water table. The plot indicates excellent visual correlation with the water table series in Figure 1, with the highest recharge rates occurring during periods when groundwater levels increased the fastest (the steepest rising parts of the graph in Figure 7). An interesting finding was that the average percolation time of the moisture front during the rainy season was always very close to the transit time of 87 d derived using the cross-correlation analysis. This is best illustrated using the precipitation data of year 2005, and the recharge data in Figure 7 of that same year. The rainfall data showed a very high peak at $t = 1491$ d (the very end of January 2005) from the start of the simulation (more than 6 and 12 cm on days 1490 and 1491, respectively). The peak of the recharge plot that year (2005) occurred on day 1480, which is 89 d after the large rainfall event. This result is very close indeed to the average transit time of 87 d found using the cross-correlation analysis.

Some caution about the aforementioned numerical results is warranted as the recharge rate depends also on the transients of the water table and the upward/downward movement of the capillary fringe during the year. One alternative boundary condition would be to use a transient condition specifying the water table vs. time consistent with the water levels shown in Figure 1, or perhaps even a constant water table. The latter case was considered by fixing the water table at a depth of 800 cm (150 cm above the monitoring point). The calculated recharge rates for this case remained very close to those in Figure 7, except that the peaks occurred slightly earlier. In particular, the peak recharge rate in 2005 occurred 65 d after the intense rainfall event at $t = 1490$ d.

We further implemented a special deep drainage lower boundary condition to account in an approximate way for regional flow effects on the recharge rate. Use of this boundary condition was motivated by an experimental study by Ernst and Feddes (1979) who established a relationship between the recharge rate and the groundwater level at a particular location (see also Hopmans and Stricker 1989). The relationship was generalized within the HYDRUS-1D software (Šimůnek et al. 2008)) to the more general boundary condition

$$q_L (t) = A \exp \left[ -B (h_L - h_{gw}) \right]$$

(15)

where $q_L$ is the imposed lower boundary flux, $h_L$ is the transient pressure head (or water table elevation) at the lower boundary, $A$ and $B$ are adjustable parameters, and
Table 3
van Genuchten–Mualem Soil Hydraulic Parameters Used for the Recharge Calculations ($\theta_r$ is the Residual Water Content, $\theta_s$ is the Saturated Water Content, $\alpha$, $n$, and $l$ Are Empirical Shape Factors, and $K_s$ is the Saturated Hydraulic Conductivity)

<table>
<thead>
<tr>
<th>Soil Depth (cm)</th>
<th>$\theta_r$ (cm$^3$/cm$^3$)</th>
<th>$\theta_s$ (cm$^3$/cm$^3$)</th>
<th>$\alpha$ (cm$^{-1}$)</th>
<th>$n$ (−)</th>
<th>$l$ (−)</th>
<th>$K_s$ (cm/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–150</td>
<td>0.039</td>
<td>0.387</td>
<td>0.0334</td>
<td>1.42</td>
<td>0.50</td>
<td>53.0</td>
</tr>
<tr>
<td>150–250</td>
<td>0.043</td>
<td>0.386</td>
<td>0.0311</td>
<td>1.40</td>
<td>0.50</td>
<td>276</td>
</tr>
<tr>
<td>250–350</td>
<td>0.046</td>
<td>0.385</td>
<td>0.0294</td>
<td>1.39</td>
<td>0.50</td>
<td>378</td>
</tr>
<tr>
<td>350–450</td>
<td>0.050</td>
<td>0.387</td>
<td>0.0257</td>
<td>1.39</td>
<td>0.50</td>
<td>436</td>
</tr>
<tr>
<td>450–550</td>
<td>0.056</td>
<td>0.386</td>
<td>0.0266</td>
<td>1.37</td>
<td>0.50</td>
<td>49.8</td>
</tr>
<tr>
<td>550–1000</td>
<td>0.057</td>
<td>0.386</td>
<td>0.0263</td>
<td>1.36</td>
<td>0.50</td>
<td>99.1</td>
</tr>
</tbody>
</table>

$h_{gw}$ is some long-term equilibrium water table position relative to the lower boundary. In this study, we considered the three parameters ($A$, $B$, and $h_{gw}$) to be completely empirical and fitted them to the available water table data.

Figure 8 shows the calculated water table position in terms of simulated pressure heads at the monitoring point, similarly as the measurements in Figure 1. Calculations used values of $−1.8$ cm/d for $A$ (the negative sign indicating a downward flux), $−0.007$/cm for $B$, and 800 cm for $h_{gw}$. Calculated water levels in Figure 8 were relatively close to the observed values in 2004 and 2005, but underpredicted the water table fluctuations in 2002 and early 2003 (between approximately 365 and 760 d).

Figure 9 shows calculated recharge rates for the deep drainage boundary condition. As compared with Figure 7, the recharge rate in Figure 9 is much less variable in time as the net downward flux now is affected by the upward and downward movement of the capillary fringe. However, the overall (cumulative) recharge rates during the 4-year simulation (2002 to 2005) for the two boundary conditions were relatively close: 137 vs. 121 cm during 4 years (34.3 vs. 30.4 cm/year). Our results indicate that predictions of the yearly averaged recharge rate using root zone modeling using a free-drainage draining lower boundary condition may give similar results as the more realistic deep drainage condition, but not necessarily of the transient recharge distribution during the year.

Concluding Remarks
Systematic monitoring of the water column made it possible to obtain a detailed time series involving both very small oscillations in amplitude (on the order of $10^{-3}$ m) related to natural phenomena such as earth tides.
and earthquakes, as well as much larger amplitudes (of approximately 2 to 3 m) related to seasonal cycles of recharge and discharge of the aquifer. Statistical analyses of the data using Fourier analysis, cross-correlation, and R/S analysis (the Hurst exponent) corroborated qualitative observations, such as the frequency of earth tides (two times per day) and seasonal oscillations (one cycle per year). Oscillations in the water table showed that the field system has a definite “memory” of previous events, thus very much reflecting the fact that the effects of future hydrologic events will be influenced by past results.

Our analysis also identified temporal relationships between rainfall and aquifer recharge, as well as the average time required for seasonal infiltration fronts to reach the aquifer (87 d in this study). The calculated average percolation time was very close to estimates obtained using numerical solutions of the Richards equation for flow in variably saturated vadose zone systems. Implementation of a special deep drainage boundary condition in the numerical solution allowed us to obtain independent estimates of the seasonal water table fluctuations, with results being very consistent with observed data. Numerical results indicated that predictions of the yearly averaged recharge rate using root zone modeling using a free-drainage draining lower boundary condition may give similar results as the more realistic deep drainage condition, but not necessarily of the transient recharge distribution during the year.

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