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Verification of numerical solutions of the Richards equation using a traveling wave solution

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Abstract

Efforts to find solutions that can be used for verification of numerical techniques for solving the Richards equation have generated a wealth of approximate and exact analytical solutions. Coefficients of this equation involve two highly non-linear functions related to the soil water potential, the unsaturated hydraulic conductivity, and the soil water content. The known exact solutions for realistic flow geometries are commonly limited to simplified descriptions of unsaturated hydraulic properties, while the approximate solutions involve various simplifications that require additional verification. We present a technique, referred to as the "launch pad" technique, which is based on the traveling wave solution to generate an exact solution of the boundary value problem for the Richards equation. The technique that is applicable to any descriptor of unsaturated hydraulic properties is illustrated on an application involving the infiltration of water into soils with properties described by Brooks–Corey and van Genuchten models. Examples of verification are presented for HYDRUS-1D, a popular numerical computer code for solving the Richards equation.

Keywords: Richards equation; Soil descriptors; Traveling wave; Numerical solution; Code verification

1. Introduction

The Richards equation for soil moisture movement occupies a very important place in modern theoretical and applied hydrology. Most commonly, tools for solving various hydrologic problems for vadose zone flows utilize numerical methods based on either the finite-difference or finite-element techniques. Recently, Scanlon et al. [18] provided an inter-comparison of the most commonly used codes in the US (HELP, HYDRUS-1D, SoilCover, SHAW, SWIM, UNSAT-H, and VS2DTI) by simulating the water balance of surficial sediments in a semiarid region. The purpose of this comparison was to assess effects of different approaches in treatment of the hydrologic assumptions for fluxes at the upper boundary (infiltration and evaporation), the lower boundary (drainage, etc.), and aquifer parameters. Vanderborght et al. [23] reviewed the verification aspects of both flow and transport model components and included several additional codes that are used in the European Union for both academic and applied purposes. A common feature of all numerical codes, whether those on the list above or others, is the lack of good benchmark tests for verification of accuracy of solution methods for transient flow conditions.

The motivation of our study was to find an exact but simple transient solution for soil moisture dynamics that can be used for any descriptor of unsaturated hydraulic properties and simple but realistic initial and boundary conditions, including relations described by tabular data. The requirement that the solution be valid for any descriptor of unsaturated hydraulic properties is especially important because various available codes use a variety of

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descriptors. It is desirable that such a solution be derivable with minimal computational effort.

During the last four decades a large number of analytical approaches have been developed (e.g., Barry et al. [2], Broadbridge and Rogers [3], Fleming et al. [8], Parlange [11,12], Philip [13–15], Pullan [16], Ross and Parlange [17], Smith et al. [21], Srivastava and Yeh [22], Warrick [24], and Witelski [25]). Most of these analytical solutions have been developed and tested for specific unsaturated hydraulic properties. Only Philip [13–15] and Ross and Parlange [17] developed solutions for general descriptors of unsaturated hydraulic properties and one-dimensional flow in a semi-infinite domain. There are also differences between these solutions.

Philip [13] provided a solution for infiltration with consatant soil moisture content at the surface into a semi-infinite soil profile. The solution is given as an expansion series for square-root-time. A disadvantage of the solution is that series converge only for finite time range [11,13] and the evaluation of series coefficients is complicated by solving an infinite system of integral differential equations. The complexity of numerical evaluation makes it prohibitive for systematic use, and in fact, characteristics of only one soil (Yolo light clay) found use in the Philip's solution [13–15], with rare exceptions [26]. However, Philip proved that after large time this solution has a self-similar front, called it "solution at infinity", and derived its analytical form [15]. The front velocity for this solution is a function of soil descriptors and boundary conditions. The transition time between small and large time solutions has not been established [15]. This solution (or so-called traveling wave [25]) can be considered as a solution of initial boundary value problem with given soil moisture content/head at $z = \pm\infty$.

Ross and Parlange [17] proposed using a self-similar solution for a specified flux at the upper boundary in a semi-infinite domain to test numerical schemes. However, the link with the Philip [15] solution has not been established. In [17], the front velocity was assigned arbitrarily for a wide range of parameters. It was also shown that surface ponding could occur for some flux values, after which this solution requires adjustments.

Traveling waves have been well known in fluid mechanics previously [1]. In hydrology, applications of traveling wave solutions are of interest in experimental studies (e.g., [19]) and infiltration theory (e.g., [6,7]). However, the practical ramifications of traveling waves and validation of numerical codes in particular have been underestimated [21].

In this article we combine approaches of Philip [15] and Ross and Parlange [17] for developing a simple and flexible method that yields an exact solution that satisfies the initial-boundary value problem for the Richards equation. This approach has the following traits: (1) it is valid for any descriptor of the hydraulic properties of unsaturated soils as in [15,17]; (2) the solution utilizes velocity c, which is defined [15] by soil properties and boundary and initial conditions, and is valid for arbitrary time; (3) the solution is presented for either specified pressure or specified flux upper boundary condition. We also present technical aspects associated with the evaluation of integrals involved in the assessment of the traveling wave profile.

These solutions were applied to soils with the most general unsaturated soil hydraulic properties used in vadoze zone hydrology such as the Brooks–Corey and van Genuchten. As an example, we use this method for verification of one of the most popular computer codes for simulation of soil moisture flow in vadose zone, HYDRUS-1D [20]. This code has been used in numerous applications; nevertheless traveling wave solutions are shown to be a valuable instrument in clarifying some aspects of the code accuracy. This means that such solutions can be a good benchmark for any test.

2. Traveling wave solution in infinite and semi-infinite spatial domains

2.1. The problem statement for semi-infinite domain

We consider vertical infiltration into an initially moist bare soil without surface saturation, using the Richards equation written in water content-based form

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial\theta}{\partial z}\right) - \frac{\partial K(\theta)}{\partial z}, \tag{1}$$

where hydraulic conductivity $K(\theta)$, diffusivity $D(\theta) = K(\theta)(\mathrm{d}h/\mathrm{d}\theta)$, and retention curve $h(\theta)$ are non-linear functions, θ is the volumetric water content, h is soil water potential, t is time, and z is a vertical coordinate oriented in the direction of gravity. Examples of the most commonly used soil moisture characteristics are given in Table 1.

The initial soil moisture distribution $\theta_0(z)$ in semi-infinite domain ($z > 0$) is non-uniform

$$\theta(z,0) = \theta_0(z), \quad z > 0, \tag{2}$$

and the transient upper boundary condition is as follows:

$$\theta(0,t) = \theta_1(t), \quad t > 0 \tag{3}$$

Our task is to find an exact solution that satisfies the initial-boundary value problem (1)–(3).

2.2. Analytical solution for infinite domain

Philip [14,15] obtained a solution for the Richards equation as a traveling wave that moves from $z = -\infty$ to $z = +\infty$ with some velocity c. The new variable

$$\theta(z,t) = \Theta(\varsigma), \text{ where } \quad \zeta = z - ct \tag{4}$$

and the wave velocity c is defined by $\theta = \theta_1$ at $z = -\infty$ and $\theta = \theta_0$ at $z = +\infty$ as follows:

$$c = \frac{K(\theta_1) - K(\theta_0)}{\theta_1 - \theta_0} \tag{5}$$

Table 1
Soil moisture characteristics

Characteristics	Formula	Parameters	
van Genuchten	$\frac{\theta(h)-\theta_r}{\theta_s-\theta_r}=S_e=\frac{1}{[1+\mid\alpha h\mid^n]^m},\quad h<0$	θ_r and θ_s	residual and saturated soil water contents, respectively
	$K(\theta)=K_s S_e^l[1-(1-S_e^{1/m})^m]^2$	K_s	saturated hydraulic conductivity
	$m=1-\frac{1}{n},\quad n>1$	S_e	effective water saturation
		α	parameter related to air-entry value
		n, m	pore size distribution indices
		l	pore connectivity parameter
		h_e	inverse of air-entry value
Brooks–Corey	$\frac{\theta(h)-\theta_r}{\theta_s-\theta_r}=S_e=\begin{cases}\lvert\frac{h}{h_e}\rvert^{-n}, & h<h_e\\ 1, & h\geqslant h_e\end{cases}$	n	pore-size distribution index
	$K(\theta)=K_s S_e^{2/n+l+2}$	l	pore-connectivity parameter

Introduction of variable ζ into (1) leads to the ordinary differential equation

$$-c\frac{d\Theta}{d\varsigma}=\frac{d}{d\varsigma}\left(D(\Theta)\frac{\partial\Theta}{\partial\varsigma}\right)-\frac{dK(\Theta)}{d\varsigma} \tag{6}$$

The exact solution in an implicit form for $\zeta(\Theta)$, can be obtained by integrating Eq. (6)

$$\begin{aligned}\varsigma&=F(\Theta)\\&=(\theta_1-\theta_0)\\&\times\int_{\theta}^{\theta_1^*}\frac{D(u)}{(\theta_1-\theta_0)K(u)-(\theta_1-u)K(\theta_0)-(u-\theta_0)K(\theta_1)}du,\\&\theta_0^*\leqslant\theta\leqslant\theta_1^*,\end{aligned} \tag{7}$$

where $\theta_1^*=\theta_1-\Delta_1$ and $\theta_0^*=\theta_0+\Delta_0$, and Δ_0 and Δ_1 are small positive values. These values regularize the integral $F(\Theta)$, because in general the integrand may have singularities at both integration limits $\theta=\theta_0$ and $\theta=\theta_1$. Eq. (7) can be integrated in a closed form for simple descriptors of $K(\theta)$ and $h(\theta)$ [17,25], but the commonly used descriptors (Table 1) must be treated numerically.

2.3. Solution evaluation

Theoretically, the wave front length (i.e., the distance between points with θ_0 and θ_1) is infinite, stretching from $z=-\infty$, where $\theta(z,t)=\theta_1$ to $z=\infty$ where $\theta(z,t)=\theta_0$ at any given moment of time. This is reflected in Eq. (7) as follows from the integrand singularities. With $\theta_0=\text{const}$ and $\theta_1=\text{const}>\theta_0$, this wetting front moves at velocity c, without changing shape. To make this solution of practical value, it is necessary to consider only the range of soil moisture contents between θ_0^* and θ_1^* to avoid integrand singularities.

We will use the term "truncated front" for this portion of the solution $\Theta(\zeta)$ or $\theta(z,t)$ lying in the range of $\theta_0^*\leqslant\theta\leqslant\theta_1^*$, where the most spatial changes in $\Theta(\zeta)$ or $\theta(z,t)$ occur. The shape of the truncated front is exact between θ_0^* and θ_1^* and one can select positive $\Delta_0=\theta_0^*-\theta_0$ and $\Delta_1=\theta_1-\theta_1^*$ as small as desired. Reducing Δ_0 and Δ_1 will increase spatial distance between the endpoints of the solution and expand the extent of a portion of the exact solution, but not change the front shape otherwise. The function $\theta(z,t)$ in domains $[\theta_0,\theta_0^*)$ and $(\theta_1^*,\theta_1]$ can be replaced by θ_0^* and θ_1^*, respectively with any desired accuracy by reducing Δ_0 and Δ_1.

We will illustrate this effect of varying parameters Δ_0 and Δ_1 in the integration of Eq. (7) using its implementation in the Maple V language [5]. Parameters for the sandy soil moisture characteristics (Table 2) were selected from the HYDRUS-1D database that provides average soil hydraulic parameters for different USDA textural classes [4].The front portions of the wave profile for both the Brooks–Corey and van Genuchten models are shown in Fig. 1 for the parameters given in Table 3 (θ_0, θ_1, and two different values of $\Delta_0=\Delta_1$). A decrease in $\Delta_0=\Delta_1$ leads to a change in corresponding ζ values and separation of the two profiles. Note that the dramatic difference in slope and curvature behaviors of profiles near $\theta=\theta_0^*$ and $\theta=\theta_1^*$ is attributed to non-linearity of the soil descriptors.

When applying this regularization procedure, the remaining portions of the traveling wave in domains $[\theta_0,\theta_0^*)$ and $(\theta_1^*,\theta_1]$ are approximated by horizontal lines starting at $\theta(z,t)=\theta_1^*$ and $\theta(z,t)=\theta_0^*$. The actual slopes of the wetting front in these sections can be determined from $(d\theta/d\zeta)=(d\zeta/d\theta)^{-1}$, which approaches zero when soil

Table 2
Parameters for soil moisture characteristics

Characteristics	θ_s	θ_r	α	h_e	n	l	K_s (cm/day)
van Genuchten	0.43	0.045	0.145	–	2.68	0.5	712.8
Brooks–Corey	0.417	0.02	–	0.138	0.592	1	504

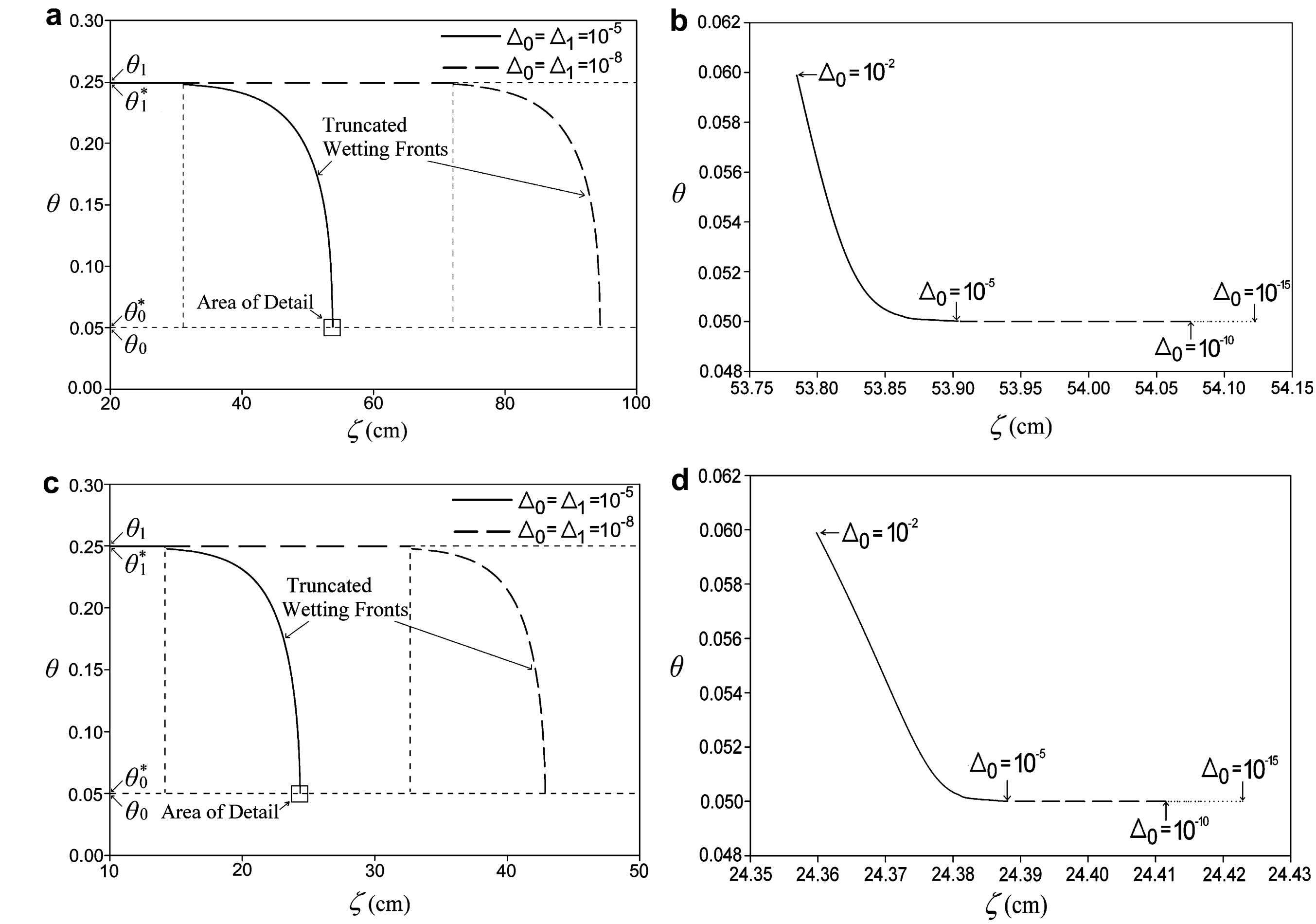

Fig. 1. Effect of regularization on computation of truncated profiles of soil moisture contents for various soil descriptors: (a) Brooks–Corey model, (b) area of detail for Brooks–Corey model, (c) van Genuchten model, (d) area of detail for van Genuchten model.

Table 3
Parameters for calculating soil moisture profiles

θ_0	θ_1	$\Delta_0=\Delta_1$	
0.05	0.25	10^{-5}	10^{-8}

moisture content approaches θ_0 or θ_1, i.e., Δ_0 or Δ_1 approaching zero. For example, slopes of the wetting front using the Brooks–Corey model are $-6.845\times10^{-2}\,\mathrm{m}^{-1}$ and $-6.853\times10^{-5}\,\mathrm{m}^{-1}$ at point $\theta=\theta_0^*$ for $\Delta_0=10^{-5}$ and 10^{-8}, respectively, and $-1.699\times10^{-4}\,\mathrm{m}^{-1}$ and $-1.699\times10^{-7}\,\mathrm{m}^{-1}$ at points $\theta=\theta_1^*$ for $\Delta_1=10^{-5}$ and 10^{-8}, respectively (see Fig. 1c). This shows that the remaining part of the solution can be replaced by $\theta\approx\theta_0$ for $\theta<\theta_0^*$ and $\theta\approx\theta_1$ for $\theta>\theta_1^*$ with any desirable accuracy by controlling Δ_0 and Δ_1, because the actual soil moisture contents θ in these parts of the profile is almost uniform.

The difficulty of using the traveling wave solution in the original form for practical purposes is that this profile is not associated with any specific moment of time [21]. However, a remarkable property of the truncated front is that it offers an opportunity to construct analytical solutions in the semi-infinite domain for specific moments of time t, if the initial front (e.g., $t=0$) position is known. Further evolution of the profile is achieved by mere translation of this portion of the front with known velocity c [7]. We will call this initial position of a truncated front a "launch pad". Naturally, the boundary condition at $z=0$ must be consistent with the front translation. Any initial position of profile, or a launch pad, at $t=0$, provides a new solution to initial-boundary value problems with any desired accuracy as will be shown below.

2.4. Selection of parameters in the analytical solution

The desired solution accuracy determines selection of Δ_0 and Δ_1. Typically, this accuracy is estimated as a fraction of the θ range. For practical purposes, $\Delta_0\sim\Delta_1=\varepsilon|\theta_0\leqslant\theta|$, where $\varepsilon=10^{-3}$–10^{-4} far exceeds the accuracy of field and laboratory measurements of θ. Decreasing Δ_0 and Δ_1 shifts and extends the length of the "profile fraction" as shown in Fig. 1d. Asymptotic expansion of the profile $\zeta=F(\Theta)$ near θ_0 and θ_1 indicates that further increase in accuracy (reduction in ε, Δ_0 and Δ_1) only weakly extends the "profile fraction'' in the parts of the profile near θ_0 and θ_1 (as $\sim\log\varepsilon$). These parts are trivial and not important for analyses or applications (see Fig. 1d).

3. Traveling wave solution in semi-infinite-domain

3.1. Solution for uniform initial soil moisture content (transient conditions at the boundary)

We consider the launch pad outside the flow domain ($z < 0$ at $t = 0$). In this case, the initial soil moisture profile is uniform

$$\theta(z,0) = \theta_0, \quad z > 0, \quad t = 0 \tag{8}$$

The truncated profile is set ''outside'' the flow domain initially, namely in the domain $-\infty < z \leqslant 0$ similarly as in Ross and Parlange [17]. As time progresses ($t_0 < t_1 < t_2 \ldots$), soil moisture content and flux at the upper boundary vary with time, and the profile ''slides'' into the domain (Fig. 2a). In our case, the constant translation velocity c is determined by soil descriptors and initial and boundary conditions in contrast with [17]. The upper transient boundary condition for soil moisture content, $\theta(0,t)$, must comply with the passage of the front through the ground surface $z = 0$. Therefore,

$$\theta(0,t) = \begin{cases} \theta_0^*, t = 0 \\ F^{-1}(-ct), \quad 0 < t < t_S \\ \theta_1^*, t > t_S \end{cases} \tag{9}$$

After time t_S, the soil moisture at the boundary becomes steady at $\theta(0,t) = \theta_1^* \approx \theta_1$, $t > t_S$ (Fig. 2b).

Estimation of time t_S can be performed using function $F(\theta)$ from Eq. (7) and its inverse function $F^{-1}(\zeta)$. Indeed, time t_S is found from relationships

$$\zeta_{|z=0} = -ct = F(\theta(0,t)), \quad -ct_S = F(\theta_1^*), \text{ and } \quad t_S = -F(\theta_1^*)/c \tag{10}$$

Note that θ_0^* and θ_1^* can approximate θ_0 and θ_1, respectively with any degree of accuracy.

The upper boundary condition can be formulated also in terms of flux. The flux at the soil surface can be derived directly from the calculated surface soil moisture content by applying the definition of flux on the soil surface and the traveling wave definition from Eq. (4)

$$q(0,t) = [(\Theta - \theta_1)K(\theta_0) - (\Theta - \theta_0)K(\theta_1) + 2(\theta_1 - \theta_0) \times K(\Theta)]_{|z=0}/(\theta_1 - \theta_0), \quad \Theta_{|z=0} = \theta(0,t) \tag{11}$$

Note, that $q(0,t) \to K(\theta_1)$ for large times because $\Theta \to \theta_1$.

3.2. Non-uniform initial soil moisture profile (constant moisture content at the boundary)

Selection of the launch pad inside the flow domain generates a solution for a specific non-uniform initial head/soil moisture distribution. In Fig. 2a, the truncated profile at $t > t_3 \approx t_S$ is practically located within the flow domain $z > 0$. This profile can be taken as another initial condition for $z > 0$ and $t = 0$, as done recently by Vanderborght et al. [23]). As time progresses, the profile translates downward with a constant velocity c.

The initial soil moisture distribution is constructed according to the traveling wave solution (Fig. 1) as described in Section 2.2, that is,

$$\theta(z,0) = \Theta(\zeta), \quad z > 0, \quad t = 0 \tag{12}$$

The upper boundary condition is then one of constant specific value of head/soil moisture contents and constant infiltration flux, and is given by

$$\theta(0,t) = \theta_1^*, \quad t > 0 \text{ or } q = K(\theta_1), \quad t > 0 \tag{13}$$

Note that θ_1^* may approximate the use of θ_1 with any desired accuracy because Δ_1 is negligibly small. With decrease in Δ_0 and Δ_1, our constructed solution becomes exact.

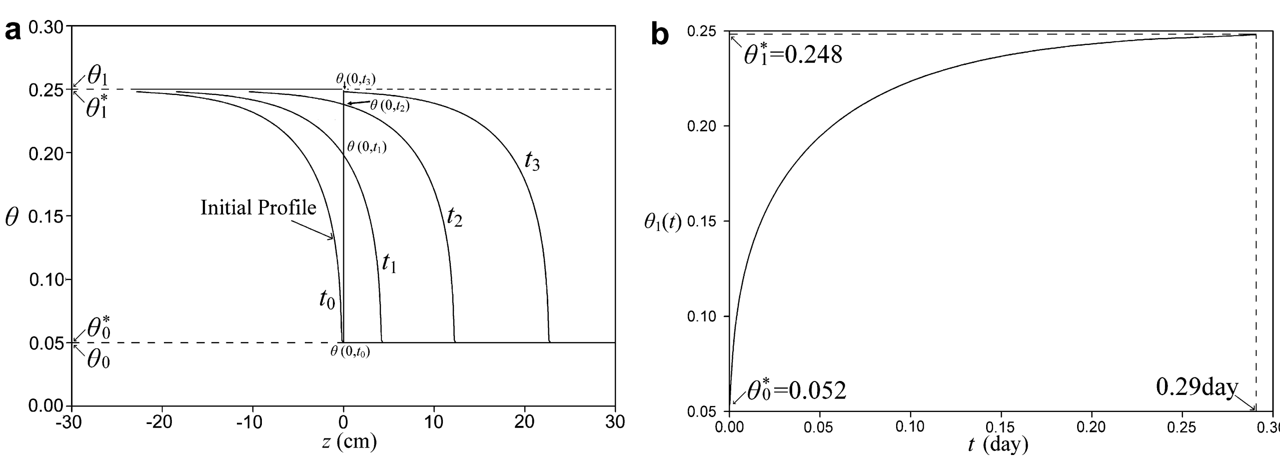

Fig. 2. Exact solution with uniform initial profile $\theta(z,0) = \theta_0$ and the passage of the front through $z = 0$ s: (a) Truncated profile is set fully outside of the flow domain, Brooks–Corey model, $z < 0$; (b) The upper transient boundary condition $\theta(0,t)$, and $t_S \approx 0.29$ day.

4. Traveling wave application for verification of a numerical code

Comparative studies [18,23] have not distinguished a distinctly superior code among the selected group of the most commonly used codes. Each computer code listed includes a complex interaction of numerical method, grid design, and input data preparation including computations or interpolation of soil descriptors for numerical simulation (e.g., [9,10]).

For example, one of the most well documented and widely used codes, HYDRUS-1D, either evaluates soil hydraulic properties directly from analytical models or by the linear interpolation from internal tables that are generated at the beginning of a numerical simulation [20]. When the internal tables are used, then HYDRUS-1D generates within a specified interval of pressure heads $[h_a, h_b]$ (or lower and upper limits of tension, LLT and ULT, respectively) a table of water content, hydraulic conductivity, and specific water capacity values from the specified set of soil parameters. One hundred logarithmically-varying intervals are used for interpolation. Values of these characteristics are then computed during the iterative solution process using linear interpolation between entries in the table. If an argument h falls outside the prescribed interval (h_a, h_b) or $h_a = h_b$, the hydraulic characteristics are evaluated directly from the hydraulic functions. The above interpolation technique was found to be much faster computationally than direct evaluation of the hydraulic functions over the entire range of pressure heads, while preserving the precision of calculated results in a majority of applications. Because of the highly nonlinear relationships between θ, h, and K, the degree of influence of LLT and ULT on simulation of the wetting front generally is not apparent.

Overall code optimization cannot be achieved by mere increase in accuracy of each code component (numerical method, discretization, and interpolation). One advantage of using analytical methods is that they provide an opportunity for sensitivity analysis and quantify influence of all steps of the numerical solution on simulation results in entirety. In this aspect, flexibility of the traveling wave solution is very attractive as a benchmark test.

Table 4
Parameters for calculating wetting fronts in initially dry soils by traveling wave solution (TWS) and HYDRUS-1D

Characteristics	θ_0 in TWS	θ_0 in HYDRUS-1D	θ_1	$\Delta_0 = \Delta_1$
van Genuchten	0.045	0.045001	0.25	10^{-5}
Brooks–Corey	0.02	0.02001	0.25	10^{-5}

The proposed traveling wave solution for uniform initial soil moisture profiles was compared with simulations performed using HYDRUS-1D. Usually, simulation of advancement of a steep wetting front into initially dry profile is a stringent test of a numerical solution of the Richards equation. Both Brooks–Corey and van Genuchten models were tested with hydraulic parameters given in Table 2 and parameters for calculating traveling wave in Table 4. Note that the initial soil moisture content was set to be slightly higher than the residual soil moisture contents for convergence. Simulations for 2-day infiltration events are presented in Fig. 3. For van Genuchten model, h_0 (corresponding to θ_0) and h_1 (corresponding to θ_1) are -1.46×10^4 cm and -8.47 cm, respectively, and for Brooks–Corey model, h_0 and h_1 are -4.25×10^8cm and -18.22 cm, respectively.

The first simulations used the default values of LLT = 10^{-6} cm and ULT = 10^4 cm in HYDRUS-1D. The results (Fig. 3) show that when comparing positions of wetting fronts at $\theta \approx \theta_0$, the simulated wetting front from HYDRUS-1D are about 6.3 and 20.1 cm/day faster than velocities of the traveling wave for the Brooks–Corey and van Genuchten models, respectively (Note that the calculated discrepancy of the position of wetting fronts is also dependent on spatial grid discretization in HYDRUS-

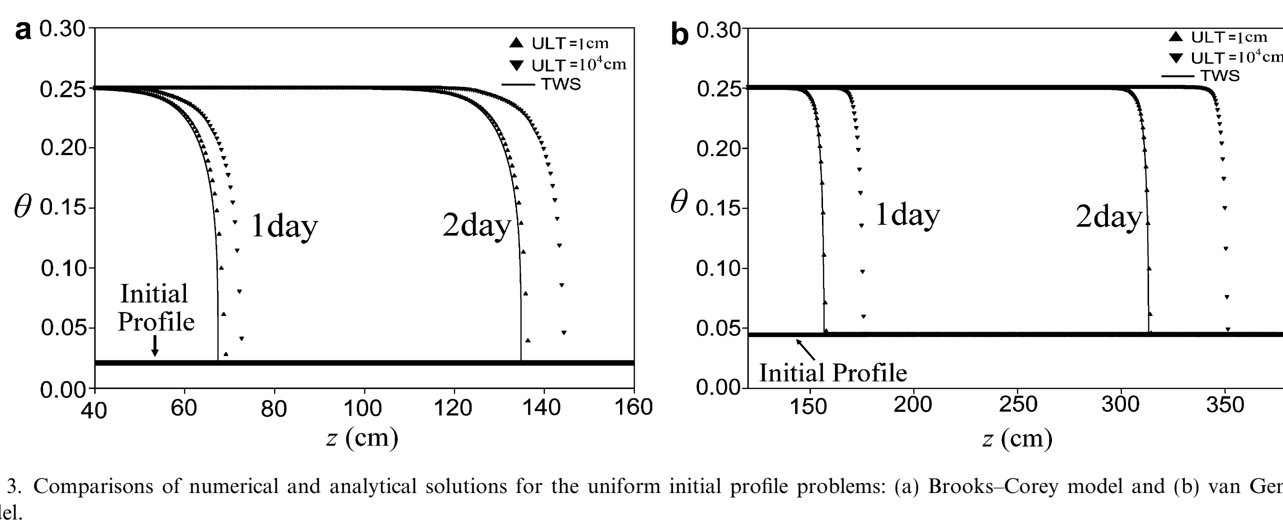

Fig. 3. Comparisons of numerical and analytical solutions for the uniform initial profile problems: (a) Brooks–Corey model and (b) van Genuchten model.

1D). When soil hydraulic properties are evaluated directly from analytical models (by specifying ULT larger than the boundary value $|h_1|$, corresponding to θ_1 (e.g., ULT = 1.0 cm), the discrepancy of wetting fronts is significantly reduced to 2.3 and 1.62 cm/day for Brooks–Corey and van Genuchten models, respectively. The results confirm the notion that evaluating the soil hydraulic properties by internal interpolation instead of direct use of the analytical functions requires a tradeoff between computational speed and accuracy.

5. Conclusions

We have revisited the exact traveling wave solutions for the Richards equation developed by Philip [14,15] and Ross and Parlange [17]. The self-similarity of traveling wave solutions allows to generate different exact solutions for infiltration problems for arbitrary finite times using the "launch pad" concept. After computing a major portion of the exact solution called a "truncated profile", the launch pad for this truncated profile can be set initially in any part of the flow domain. If the launch pad for the truncated profile is initially located fully within the flow domain ($z > 0$), the solution for infiltration with constant moisture content and flux at the boundary is generated. If the launch pad for the truncated profile is located outside of the domain ($z < 0$), this approach generates a solution for infiltration into the initially uniform soil moisture profile with a certain transient boundary condition, when the surface soil moisture content is consistent with the traveling wave profile. We presented the procedure for evaluating the truncated profile and analytical expression of this time-dependent boundary condition. This approach is valid for arbitrary soil characteristics and for specific soil-moisture content-based or flux-based boundary conditions at the surface.

The practical use of the launch pad technique in semi-infinite vertical soil profile was illustrated by assessment of the effects of computational parameters of a commonly used code, HYDRUS-1D for Brooks–Corey and van Genuchten soil descriptors. Overall verification of numerical codes with interplay of numerical method, discretization, and interpolation can be facilitated by using flexible traveling wave solutions as benchmark tests. Such an approach can be utilized for numerical codes with any realizable set of unsaturated hydraulic property descriptors.

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