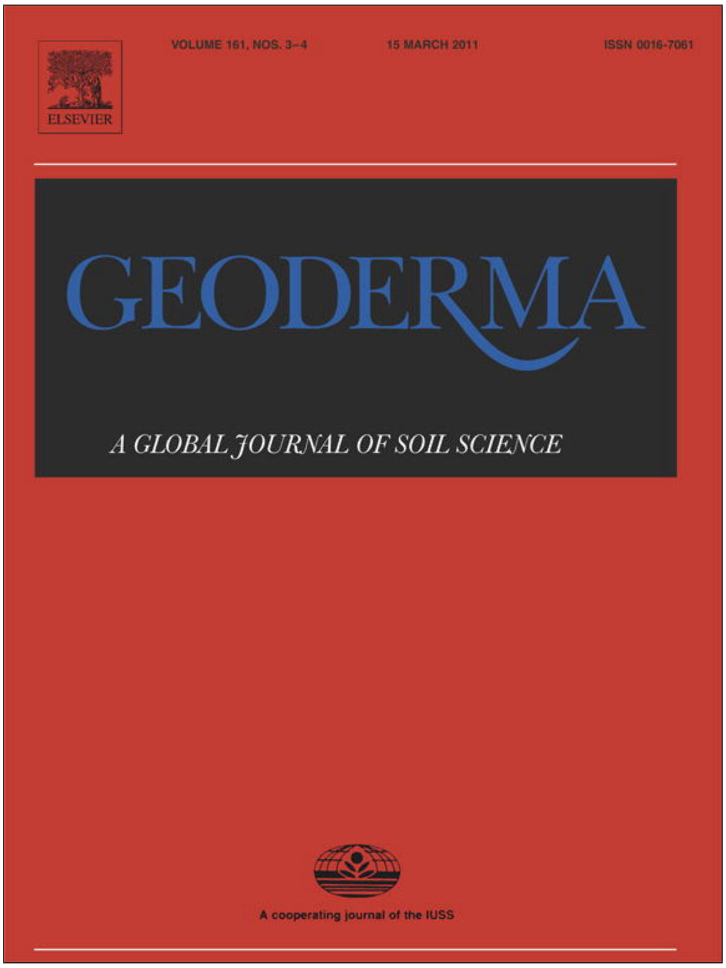
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Determining the influence of stones on hydraulic conductivity of saturated soils using numerical method

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ABSTRACT

Mountainous soils usually contain a large number of rock fragments (particle diameter>2 mm), which influence soil hydraulic and retention properties. Data characterizing the properties of these soils usually describe only the fine earth (particle diameter<2 mm). To quantitatively describe soil water movement in stony soils, their most important characteristic, i.e., the effective saturated hydraulic conductivity (K_{se}), must be known. The objective of this study was to use a numerical method for estimating the saturated soil hydraulic conductivity that depends on relative stone content (stoniness), and sizes and shapes of stony parts. The method is based on a numerical version of the classical experiment of Darcy. The steady-state water flow under a unit hydraulic gradient through hypothetical soils containing stones was simulated using the two-dimensional simulation model HYDRUS-2D. Four soil textural types were considered. Special attention was paid to the moraine soil from the FIRE site in High Tatras. A relationship between the relative saturated hydraulic conductivity, K_r, and the relative stone content, R_v, was derived. $K_r(R_v)$ function is decreasing slower for larger stones. Numerical results were used to propose an empirical equation to estimate K_r of soils containing rock fragments of a spherical shape of various diameters.

1. Introduction

A soil, often described as a weathered upper layer of the Earth's surface (Hillel, 1971), is the source of water and nutrients, and an ideal growth environment for plants (Hraško & Bedrna, 1998; Jury & Horton, 2003). Soil particles, which are a product of weathering, have many different sizes and shapes. It is generally accepted to divide soil particles into two fractions: the fine earth with a diameter of particles less than 2 mm, and the rock fragments with particles larger than 2 mm.

Water moves through pores, the size and shape of which depend on sizes, shapes and arrangements of solid particles. Soil water movement is usually described on a macroscopic scale, since for the quantitative description of water movement in pores (a microscopic approach) one would need to describe the geometry of porous pathways. The most frequently used macroscopic equation describing soil water movement in homogeneous (or layered) soils is the Richards equation (Richards, 1931). This equation needs the "macroscopic" soil hydraulic characteristics as an input.

The soil hydraulic properties (i.e., the hydraulic conductivity function and the retention curve) are in the macroscopic approach characterized for the so-called representative elementary volume (REV) of the soil (Bear, 1972; Nordahl & Ringrose, 2008). REV is such a volume of the porous media (soil), that hydraulic characteristics would change when a smaller soil volume was used for their determination, and would remain unchanged when a larger soil volume was used. REV has to, therefore, contain enough pores and soil particles to represent the soil macroscopically, so that the Darcy's approach can be used.

Baker and Bouma (1976), Lichner (1994), and Kutilek and Nielsen (1994), recommended to define REV as a soil sample with its cross-section containing no less than 20 elementary units of the soil structure (particles, peds, or macropores). A cross-sectional area corresponding to REV is usually between 10 cm^2 and 1 m^2 (Kutílek & Nielsen, 1995). Current methods for determining basic soil texture and soil hydraulic characteristics usually use soil samples with a volume of 100 cm^3 (Kopecky cylinders). Kopecky cylinders fulfill the above requirement only for the fine soil fraction. To estimate hydraulic characteristics of soils containing rock fragments, it is necessary to either use larger soil samples, to measure hydraulic characteristics in the field, or to develop new methods for their determination. For soils containing stones a soil sample of 1 m^3 volume would be needed.

Since arable soils, after decades or centuries of stone removal, are usually composed mainly of fine earth, standard procedures can be applied to evaluate their hydraulic characteristics. On the other hand, mountainous (or forest) soils usually also contain coarse rock fragments (Mehuys, et al., 1975). At the site under consideration

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(FIRE site in High Tatras), at a depth of 50 cm below the soil surface, up to 49% of the soil volume consists of rock fragments larger than 1 cm in diameter (Novák et al., 2008). Such high relative content of stones significantly influences both soil water retention and hydraulic conductivity and their determination thus needs a special attention.

The first objective of this study is to propose a method for determining the effective saturated hydraulic conductivity K_{se} of soils containing significant amount of stone fragments, and to compare obtained results with those for the fine earth of the same soil. The second objective is to compare Eqs. (1)–(5) to estimate effective saturated hydraulic conductivity of stony soils (K_{se}) with the proposed method.

2. Theory and methods

2.1. Saturated hydraulic conductivity of stony soils

Multiple equations have been derived to calculate the saturated hydraulic conductivity of soils containing rock fragments K_{se}, LT^{-1}. Equations for K_{se} calculation developed by Corring and Churchill (1961) and Vasiliev and Tanaeva (1971) are based on the analytical solution of heat flux in dispersed systems.

The hydraulic conductivity of the fine earth saturated with water K_s, LT^{-1} is constant and the hydraulic conductivity of rock is approximately zero (it can be neglected when compared with K_s of the fine earth). As a result, equations derived by Corring and Churchill (1961) can be written for stony soils as follows:

$$K_{se} = K_s\left(\frac{2(1-R_v)}{2+R_v}\right) \tag{1}$$

$$K_{se} = K_s\left(\frac{1-R_v}{1+R_v}\right) \tag{2}$$

where R_v is the relative volume of the dispersed phase, i.e., the ratio of the volumetric stone content (V_r) and the soil volume (V). This ratio is sometimes called stoniness. Eq. (1) was derived for spherical stones and Eq. (2) for cylindrical stones. The orientation of the cylindrical stones with flow direction is not mentioned. Peck and Watson (1979) (acc. to Bagarello & Iovino, 2007) derived an equation identical with Eq. (1).

Bouwer and Rice (1984) intuitively derived an equation for calculation of K_{se} in the form of the relative saturated hydraulic conductivity K_r:

$$K_r = \frac{K_{se}}{K_s} = \frac{e_{se}}{e_s} \tag{3}$$

where e_{se} and e_s are void ratios of the soil with stones and of the fine soil fraction alone, respectively. Brakensiek et al. (1986) showed, that Eq. (3) can be approximated to:

$$K_r = \frac{K_{se}}{K_s} = 1-R_w \tag{4}$$

where R_w ($=m_r/m$) is the relative stone content (stoniness), expressed in mass units; m_r is the mass of stones in soil, and m is the mass of soil with stones in a given volume of soil.

3. Method

As it was stated above, the second objective of this work will be achieved by numerically simulating steady-state water flow under a unit hydraulic gradient through hypothetical soils containing stones using the two-dimensional simulation model HYDRUS-2D (Šimůnek et al., 2006, 2008). As will be shown below unsaturated zones develop below the stones depending on their sizes and thus variably-saturated

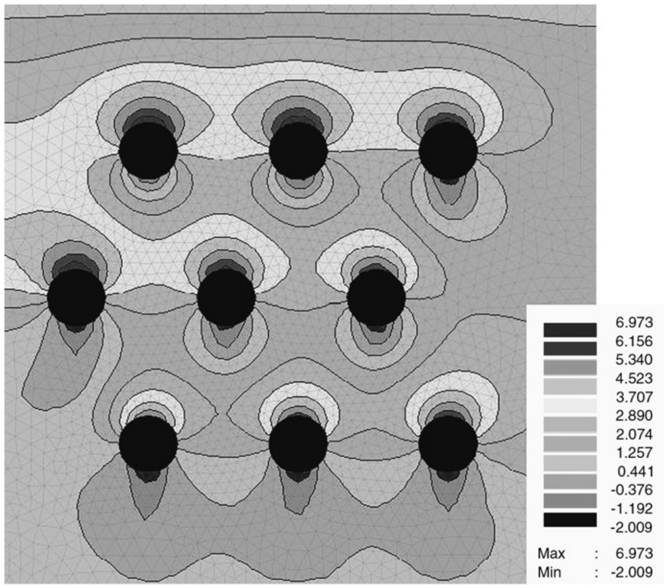

Fig. 1. Distribution of water pressure heads during steady-state water flow through the soil containing stones (calculated using HYDRUS-2D). A cross-sectional area of the soil sample is 1 m^2, a diameter of dispersed stones is 10 cm, and R_v = 0.071. Steady-state water pressure heads ranged from −2 to 6 cm.

flow model is needed to solve this problem. HYDRUS-2D is a computer model that solves numerically highly nonlinear Richards equation for a selected set of soil hydraulic functions. Note that the Richards equation can be linearized using the Kirchhoff transformation with an exponential conductivity function and then analytical solutions can be derived for relatively simple geometries and homogeneous (e.g., Kacimov, 2000; Philip, 1968; Raats, 1971) or heterogeneous (Bakker & Nieber, 2004; Sviercoski et al., 2009; Warrick and Knight, 2002) soils. For example, Warrick and Knight (2002) presented an analytic element solution for a single embedded cylindrical inhomogeneity having a different saturated hydraulic conductivity than the surrounding uniform soil. However, for soils characterized using soil hydraulic functions of a general shape (e.g., van Genuchten, 1980), the effect of embedded inhomogeneities on vadose zone flow and transport can be quantified only using numerical solutions of the Richards equation (e.g., Ju and Kung, 1993, 1997).

Water flow in a vertical cross-section (1*1 m) of a hypothetical soil sample of a 1-m^3 volume (1*1*1 m) with impermeable side faces was simulated using HYDRUS-2D (Šimůnek et al., 2006, 2008). Stones were approximated in this cross-sectional area as circles, representing thus impermeable cylinders in the full three-dimensional sample.

As 1 cm of water head was applied at both upper and lower surfaces, the pressure water head difference between the upper and lower boundaries was exactly 1 m. For a unit pressure head gradient the rate of water flow equals to the saturated hydraulic conductivity K_{se}.

The stone content (stoniness) of this virtual soil sample differed between individual experiments. First, water flow was simulated through a homogeneous soil consisting only of the fine earth fraction

Table 1
A number (n) and a diameter (D) of spherical stones, and the relative stone content R_v in hypothetical stony soil samples used in simulations; d_1 is a diameter of a single (integral) stone sphere corresponding to a particular R_v.

n	D [cm]	R_v[cm^3 cm^{-3}]	n	d_i [cm]	R_v[cm^3 cm^{-3}]
9	10	0.071	1	24	0.045
20		0.157		50.5	0.2
30		0.235		61.8	0.3
40		0.314		80	0.5

Table 2
Textural characteristics of the soil fine earth at the FIRE site (using data of Bärwolf, 2006).

Depth cm	Particles diameter range [mm]	Percentage of soil fraction (averages)	Percentage of soil fraction (range)	Soil fraction
0-110	>0.05	60	55–65	Sand
	0.05–0.002	28	26–29	Silt
	<0.002	12	7–15	Clay

of the stony soil. Here, the resulting final water flux must be equal to the saturated hydraulic conductivity of the fine earth (K_s). Next, numerical experiments were performed for relative volumes of stones equal to 0.071, 0.157, 0.235, and 0.31. Spherical stones, represented by impermeable circles with a diameter of 10 cm were chosen to represent stones in the soil profile. This diameter was close to the average diameter found at the FIRE field site. To cover a wide range of real stones, numerical experiments were conducted also for stones with a diameter of 5, 10, and 20 cm, and for different stoniness.

Stones as impermeable circles were regularly distributed in the virtual soil sample to obtain required relative volumes. To evaluate the influence of the different stone size on K_{se} at the same relative stone content, the stone contents (0.09, 0.2, 0.3, and 0.4) of soil samples with stones of diameter 10 cm were reproduced with one big stone of the spherical (circular in 2D) shape in the second set of experiments. Two different values of K_{se} for the same relative stone content R_v were thus obtained.

4. Results and discussion

An example of spatial distributions of stones and pressure heads in one simulation is shown in Fig. 1. Note positive pressure heads on the top of stones and negative pressure heads below the stones. Numerical solution of this flow problem is required because of these negative pressure head zones below stones. If the entire soil profile was saturated, including areas below stones, the highly nonlinear Richards equation would simplify into a linear Laplace equation, which could be solved using various analytical or semianalytical methods.

Characteristics of rock fragments (stones) and virtual soil samples used in numerical experiments are given in Table 1. Water flow simulations were performed for four types of the fine earth fraction, to estimate their influence on the effective hydraulic conductivity of stony soils. The particle distribution of the fine earth fraction from the moraine soil sample from the FIRE site in High Tatras is given in Table 2. This soil represented in this study the actual soil. Additional simulations were carried out for three textures, i.e., sandy loam, loam, and clay, that were taken from the HYDRUS database. Soil hydraulic parameters of the four soils considered in this study are given in Table 3.

Results of water flow simulations of steady-state water flow for soil samples with different stone contents are presented in Fig. 2. Fig. 2 shows that there is an inverse relationship between the effective hydraulic conductivity, K_{se}, and the relative stone content of the soil, R_v, for all four studied soils. Fig. 2 also shows that the effective hydraulic conductivity of a soil from the FIRE site with a given R_v is lower when the soil contains a single "large" stone than when it contains multiple 10-cm diameter stones. Similar results were obtained also for a clay soil. For the other two textures, i.e., loam and sandy loam, the effective saturated soil hydraulic conductivities K_{se} were similar for soils with a single stone or multiple stones with the same stoniness. The differences in K_r for different stone diameter are very small, which simplifies the calculation of K_r in this range of R_v.

Table 3
Soil hydraulic characteristics of the soil fine earth of four soils with different texture. θ_r and θ_s are the residual and the saturated soil water contents, respectively, α and n are the van Genuchten (1980) shape parameters, and K_s is the saturated hydraulic conductivity of the fine earth.

Soil	θ_r	θ_s	α [cm^{-1}]	n	K_s [cm min^{-1}]
Sandy loam	0.065	0.410	0.075	1.890	0.074
Loam	0.078	0.430	0.036	1.560	0.017
Clay	0.068	0.380	0.008	1.090	0.003
Loamy sand (FIRE)	0.040	0.622	0.106	1.230	0.465

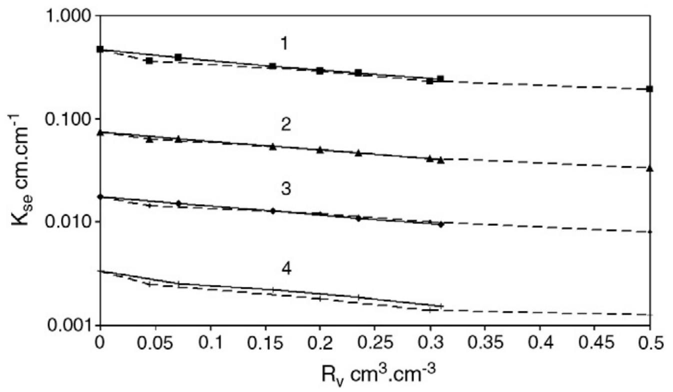

Fig. 2. Effective saturated hydraulic conductivities K_{se} as a function of the relative stone content R_v calculated using numerical experiments. Loamy sand from the FIRE site in High Tatras (1), sandy loam (2), loam (3), and clay (4). Full lines represent soils with uniformly dispersed stones of a 10-cm diameter, dashed lines represent soils with a single large stone and the same R_v. Soil characteristics are given in Tables 1 and 3.

Results of numerical experiments with soils containing stones of various sizes are presented in Fig. 3. Note that the decrease of K_{se} with increasing R_v is different for soils with stones of different sizes. The effective hydraulic conductivity K_{se} for the soil containing the "smallest" stones with a diameter of 5 cm decreases faster, compared to the soil containing stones with "bigger" diameters. It can be concluded that the bigger the stones in the soil, the slower the decrease of its effective saturated hydraulic conductivity with increasing relative stoniness. Differences in the $K_{se}(R_v)$ relationship for different sizes of rock fragments can be explained by different hydraulic losses due to bypassing of the stones. The smaller the stones, the higher the resistance to flow at a given stoniness, because for the same R_v the number of small stones increases.

Fig. 4 shows a comparison of the $K_r(R_v)$ relationships obtained using Eqs. (1), (2), and (4) with those obtained using numerical experiments for loamy sand from the FIRE site in High Tatras with

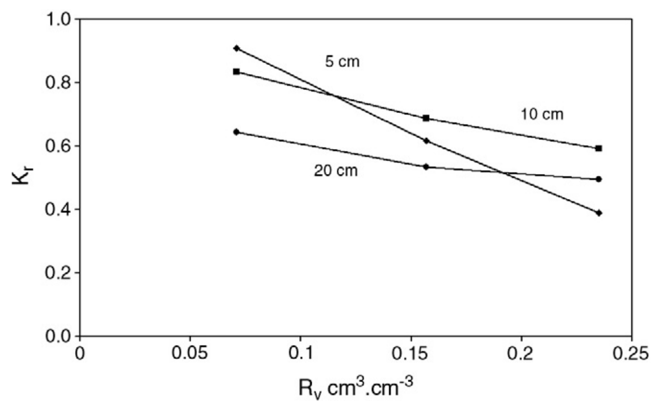

Fig. 3. Relative saturated soil hydraulic conductivities K_r as a function of the relative stone content R_v (calculated using HYDRUS-2D). Loamy sand from the FIRE site in High Tatras with three different stone diameters (5, 10, and 20 cm) was used.

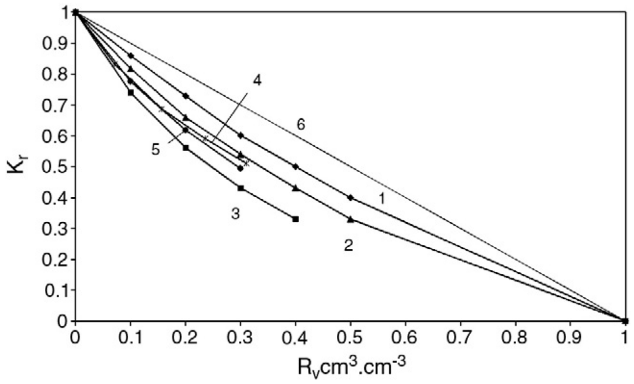

Fig. 4. Relative saturated soil hydraulic conductivities K_r as a function of the relative stone content R_v (calculated using HYDRUS-2D). The loamy sand from the FIRE site with uniformly dispersed stones of a 10-cm diameter (curve 4), a single big stone and the same R_v (curve 5). Other $K_r(R_v)$ functions were calculated using Eq. (1) (curve 1), Eq. (2) (curve 2), Eq. (4) (curve 3), and a linear relationship (line 6). Soil hydraulic parameters used in calculations are given in Table 3.

either multiple regularly distributed stones with a diameter of 10 cm or a single big stone. The $K_r(R_v)$ relationships obtained with numerical experiments are similar to those evaluated using analytical solutions of the heat transport equation and applied to porous media. While the solutions would be identical if the soil profiles were fully saturated, for which the nonlinear Richards equation would convert into a linear Laplace equation, some deviations between the two solutions could be due to unsaturated zones that developed in the flow “shadows” of the stones. As the numerical solution of the Richards equation implemented in HYDRUS-2D can account for the effects of these zones, it should provide more realistic results. Analytical solutions of the linear Fourier heat transport equation with a constant thermal conductivity are thus only approximations. Notice that Corring and Churchill's (1961) Eqs. (1) and (2) obtained for heat transport through a conductive medium with dispersed bodies of spherical and cylindrical shapes, respectively, produce lower values of K_r for cylindrical impermeable objects.

Sauer and Logsdon (2002) identified, using infiltration tests, a small increase of K_{se} with an increase of the stone content in two soil types at the pressure head of −12 cm. In several other cases K_{se} did not respond to changes in R_v. Sauer and Logsdon (2002) estimated the $K_{se}(R_v)$ relationship using infiltration tests and some of their results can likely be explained by spatial variability of both variables. A possible explanation for an K_{se} increase with increasing R_v at positive pressure heads are the so-called lacunar pores, i.e., stable pores existing at the interface between stones and fine soil particles. Such relatively stable macropores preserve their size and shape, and their volume is proportional to the stone content (Fiés et al., 2002). These types of pore are specific for some types of stony soils. In our case (a loamy sand with stones at the FIRE site), the soil was classified as the moraine soil and no macropores at the interfaces between stones and the soil matrix were observed. Therefore, estimated K_r values decrease with an increase in R_v.

The $K_r(R_v)$ relationship is in general nonlinear. However, as a first approximation for calculation of K_r, when there are uncertainties in the identification of stone properties (sizes, shapes and their distribution), it can be expressed using a linear equation as follows:

$$K_r = 1 - a\,R_v. \qquad (5)$$

Values of the coefficient a were estimated to be between 1.1 and 1.32 for soils containing stones with a diameter of 10 cm. The lower value was estimated for sandy clay, the higher value for clay (Table 4).

Fine textured soils containing single stones can be characterized using a higher coefficient a, generally within the range from 1.32 to 1.93. Coefficients a for all four analyzed soil types and multiple and single stones are given in Table 4. Eq. (5) can be used in the range of $0 < \leq 0.4$. It can be seen in Fig. 2, Eq. (5) can really cover K_r (R_v) linear relationship in the R_v range, under consideration.

Numerical results indicated several conclusions: a) $K_r(R_v)$ relationships are nonlinear, b) K_r decreases with the relative stoniness, and c) $K_r(R_v)$ decreases faster for stones with a smaller diameter.

Table 4
Parameters a in Eq. (7) for soils with different texture and different stone sizes.

Soil type	a (stone diameter = 10 cm)	a (a single stone)
Loamy sand (FIRE)	1.2	1.66
Sandy loam	1.1	1.32
Loam	1.1	1.32
Clay	1.32	1.93

5. Conclusions

Hydraulic conductivities of saturated stony soils (K_{se}) decrease nonlinearly as a function of the relative stone content R_v. Their values are in general lower than those expressed by the linear function (5) (line 6, Fig. 4), because of the additional hydraulic resistance of the rock fragments to flow. K_{se} decreases with an increase in stoniness (R_v), and is additionally also sensitive to the stone size. K_{se} (R_v), decreases slower with increasing stone diameter. A relationship between the relative saturated hydraulic conductivity, K_r, and the relative stone content R_v, can be approximately expressed by the linear Eq. (5). The coefficient a varies from 1.1 to 1.32 for spherical stones of a 10-cm diameter, depending on the soil matrix. Eq. (5) can be used to estimate the relative saturated hydraulic conductivity of stony soils for soils with different texture as an alternative to Eqs. (1), (4), which do not consider soil texture and stone diameter.

The method presented here can be used to calculate K_{se} of stony soils with various stoniness, dimensions and shapes of stones, using coefficients a (Table 4).

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