

## ESTIMATING SOIL HYDRAULIC PROPERTIES FROM TRANSIENT CONE PERMEAMETER DATA

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We present here a method for determining the hydraulic properties of unstratified soil via inverse modeling. A modified cone penetrometer, called a cone permeameter, has been designed to inject water into the soil through a screen and to measure the progress of the wetting front with two tensionmeter rings positioned above the screen. Cumulative inflow and pressure head readings are analyzed to obtain estimates of the hydraulic parameters describing the hydraulic conductivity curve  $K(\theta)$  and the soil-moisture characteristic curve,  $\theta(\psi)$ . Two sets of cone permeameter tests, which differ in the method of placement in the soil, are discussed. In the first case, the cone permeameter was buried; in the second case, it was placed by direct push to the testing depth. Optimizations for various sets of unknown parameters, with and without additional soil moisture information, were performed. The effects of applied pressure head at the source and of initial pressure readings on the resulting hydraulic parameters are investigated. The optimization results are compared with the results of standard laboratory and field methods. The saturated hydraulic conductivity was well estimated. For cases in which the saturated moisture content was estimated or fixed near the laboratory-derived value, the soil-moisture characteristic curves were between the wetting and drying curves obtained from other standard methods. (Soil Science 1998;163:436-453)

**Key words:** Soil hydraulic properties, parameter estimation, cone penetration testing, unstratified hydraulic conductivity, inverse modeling.

**I**N the past, solution of variably saturated flow problems required use of analytical or quasi-analytical equations and adherence to strict boundary and initial conditions. Today a wide variety of saturated-unsaturated flow codes exists for the numerical solution of such problems. Use of these codes, however, requires inputs of soil hydraulic properties. As a result, accurate descriptions of the soil-moisture characteristic and hydraulic conductivity curves,  $\theta(\psi)$  and  $K(\psi)$ , are particularly important. Although these soil properties can be determined in the laboratory, in situ methods that better reflect field behavior are often preferred.

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In situ direct measurement of point data on the soil-moisture characteristic curve,  $\theta(\psi)$ , and/or the hydraulic conductivity curve,  $K(\psi)$ , may be obtained using a number of methods, including the instantaneous profile, crust, and infiltration methods (Klute and Dirksen 1986; Benson and Grubb 1997). Although the concept is relatively simple, these direct measurement methods have a number of limitations that restrict their use in practice. For example, most methods are very time consuming. Methods requiring repeated steady-state flow situations or other equilibrium conditions are also tedious, whereas linearizations and other approximations or interpolations to allow analytic or semi-analytic inversions of the flow equation introduce additional errors. Finally, information about the uncertainty in the estimated hydraulic parameters is not readily obtained using direct inversion methods.

Parameter optimization is an indirect approach that makes it possible to obtain  $K(\psi)$  and

$\theta(\psi)$  simultaneously from transient flow data (Kool et al. 1987). In this case, a flow event is modeled with an appropriate governing equation and analytical expressions of  $K(\psi)$  and  $\theta(\psi)$ . The unknown parameters of  $K(\psi)$  and  $\theta(\psi)$  are obtained by minimization of an objective function describing the differences between some measured flow variables and those simulated with a numerical flow code. This methodology was applied originally to laboratory one-step column outflow data (Kool et al. 1985; Parker et al. 1985; van Dam et al. 1992). To reduce the problem of nonuniqueness of the optimized parameters (Carrera and Neuman 1986; Toorman et al. 1992), investigated the use of pressure head data in addition to one-step outflow data. Van Dam et al. (1994) showed that the transient outflow data alone obtained from a modified multi-step column outflow experiment are sufficient to obtain unique estimates of the soil hydraulic properties. Finally Eching and Hopmans (1993) and Eching et al. (1994) presented an improved parameter estimation technique based on analysis of multi-step outflow volume and pressure head data. Parameter estimation has also been used with data obtained with the evaporation method described by Wind (1968) (see, for example, Santini et al. (1995); Ciolano and Romano (1995); Šimůnek et al. (1998a)). In this case, the upper boundary flux calculated from the decrease of weight of the soil sample with time is used as the upper boundary condition. Transient pressure head data from at least one location in a soil core and the total water volume at an arbitrary time (usually at the beginning or end of the experiment) are included in the formulation of the objective function. All of these laboratory methods provide information about the drying branches of the soil-moisture characteristics.

For field determination of the wetting branches of the soil hydraulic properties, Russo et al. (1991) and Bohne et al. (1993) studied the application of parameter estimation methods to ponded infiltration flow data. Šimůnek and van Genuchten (1996) analyzed hypothetical tension disc infiltrometer flow data and showed that additional pressure head data are necessary for numerical convergence and uniqueness of the optimized parameters. As with one-step and multi-step column outflow experiments, they found later that only inflow data obtained from a multiple tension disc infiltrometer experiment with additional initial and final moisture contents from below the disc are sufficient to obtain unique estimates of the soil hydraulic properties (Šimůnek and van

Genuchten 1997). Another method for gaining information about the drying branches of the soil hydraulic properties via multi-step soil water extraction and parameter optimization was developed by Inoue et al. (1995). They extracted water with a ceramic soil solution sampler at sequentially applied underpressure (vacuum) steps and analyzed cumulative extraction volume data along with pressure heads measured with tensionmeters at three distances from the sampler.

The field methods described above are applicable only in the near surface. Grubb (1993, 1996) proposed a new cone penetrometer tool, a cone permeameter, and use of parameter optimization to estimate soil hydraulic properties at depth. A prototype cone permeameter was developed further by Leonard (1997) (Figs. 1 and 2). A detailed description of the prototype, its use under saturated and unsaturated conditions, as well as the typical courses of such experiments were presented by Grubb et al. (1998). For each test, the cone permeameter is placed in the soil, and a constant head is then applied to the 5-cm-long screen. Cumulative inflow volume is determined from scale readings of the mass of water removed from the source. Progress of the wetting front is measured with tensionmeter rings 5 and 9 cm above the screen. After the water supply valve is closed, the tensionmeters monitor the redistribution of water in the soil profile.

Here we present observed data and numerical analysis of data collected during the wetting periods of tests performed by Leonard (1997) in a laboratory aquifer system. The numerical inverse code of Šimůnek and van Genuchten (1996) was used to predict the soil hydraulic properties of the tested soil from cumulative inflow volume and pressure head data. It is obvious that the way in which the cone permeameter is placed in the soil can influence the hydraulic property values determined by parameter optimization. To study this issue, we selected two sets of tests. The first set of tests was performed with the permeameter after it was buried in the soil, and a second set of tests was conducted after direct push to the testing depth. Next, we investigated the impact of different applied pressure heads and initial conditions on the courses of the experiments and, consequently, the parameter estimates for both sets of tests. Grubb (1996) showed that inverse analysis of data obtained with the cone permeameter provides good estimates of  $K_s$  and  $\alpha$  values, but that the method is less able to predict the real values of  $\theta$  and  $n$  (see Eqs. (4) and (5) below). Therefore, to improve pa-

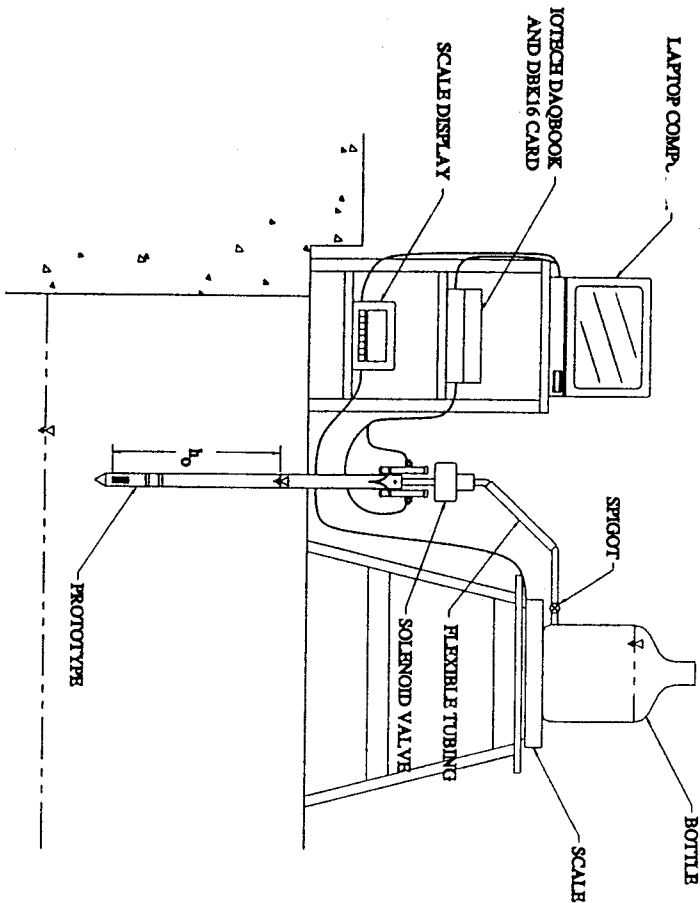


Fig. 1. Cone permeameter test set up.

parameter estimates, we included initial and final soil moisture data in the analysis, as recommended by Šimůnek and van Genuchten (1997).

**THEORY**

HYDRUS-2D (Šimůnek et al. 1996) is used to simulate the cone permeameter test in unsaturated soil with the finite element mesh shown in Fig. 3. The governing flow equation for radially symmetric isothermal Darcian flow in an isotropic, rigid, porous medium, assuming that the air phase plays an insignificant role in the liquid flow process, is (Richards 1931):

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r K \frac{\partial h}{\partial r} \right] + \frac{\partial}{\partial z} \left[ K \left( \frac{\partial h}{\partial z} + 1 \right) \right] = \frac{\partial \theta}{\partial t} \quad (1)$$

where  $r$  is the radial coordinate ( $L$ ),  $z$  is the vertical coordinate positive upward ( $L$ ),  $t$  is the time ( $T$ ),  $h$  is the pore water pressure head ( $L$ ), and  $K$  ( $L/T^2$ ) and  $\theta$  ( $L^3/L^3$ ) are the hydraulic conductivity and volumetric moisture content, respectively. Equation (1) is solved numerically for the following boundary and initial conditions:

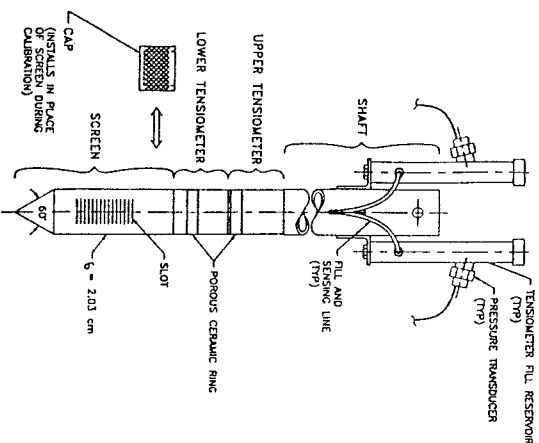


Fig. 2. Prototype cone permeameter.

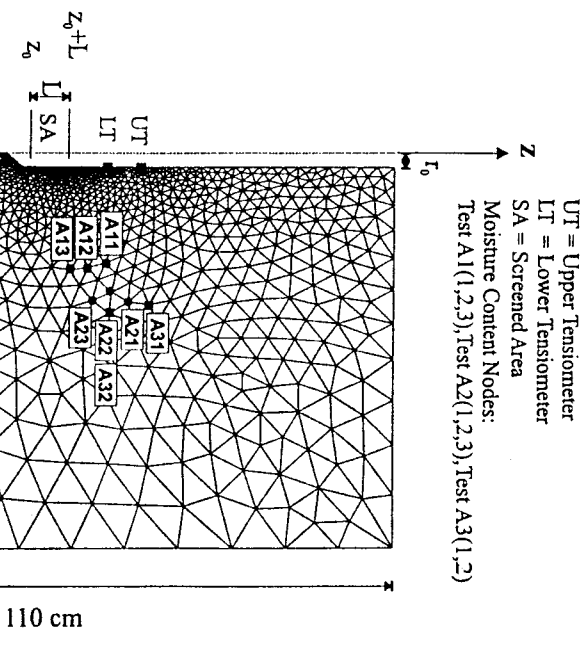


Fig. 3. Finite element mesh used for modeling cone permeameter test and location of moisture content samples.

$$h(r, z, t) = h_0 - (z - z_0) \quad r = r_0, z_0 \leq z \leq z_0 + L \quad (2)$$

the source that they do not to influence the solution and are defined as no-flow boundaries.

where  $h_0$  is the initial pressure head in the soil ( $L$ ),  $h_0$  is the supply pressure head imposed at the bottom of the screen ( $L$ ),  $z_0$  is the coordinate of bottom of the screen ( $L$ ),  $L$  is the length of the screen, and  $r_0$  is the radius of the screen ( $L$ ). Exterior boundaries are located far enough from

The van Genuchten (1980) expressions for moisture content and hydraulic conductivity,  $\theta(h)$  and  $K(\theta)$ , are used in this work:

$$\theta = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \frac{1}{1 + |\alpha|h^n} \quad h < 0 \quad (4)$$

UT = Upper Tensiometer  
 LT = Lower Tensiometer  
 SA = Screened Area  
 Moisture Content Nodes:  
 Test A1(1,2,3), Test A2(1,2,3), Test A3(1,2)

$$K(\theta) = K_h (1 - \theta^m)^{-1} \quad h < 0 \quad (5)$$

where  $\theta_s$  is the effective water content ( $-$ ),  $K_h$  is the saturated hydraulic conductivity ( $L/T$ ),  $\theta_r$  and  $\theta_s$  are the residual and saturated water contents ( $L/L$ ), respectively,  $n$  is the pore-connectivity parameter ( $-$ ), and  $\alpha$  ( $L^{-1}$ ),  $n$  and  $m$  ( $= 1 - 1/n$ ) are empirical parameters. The predictive  $K(\theta)$  model is based on the capillary theory of Mualem (1976) in conjunction with Eq. (4). The pore-connectivity parameter,  $l$ , in the hydraulic conductivity function was estimated by Mualem (1976) to be about 0.5 for many soils. The hydraulic characteristics defined by Eqs. (4) and (5) contain five unknown parameters:  $K_h$ ,  $\theta_r$ ,  $\theta_s$ ,  $\alpha$ , and  $n$ .

The objective function,  $\Phi$ , minimized during the parameter optimization process, is (Šimůnek et al. 1998b):

$$\begin{aligned} \Phi(u, q, p) = & \sum_{j=1}^m \nu_j \sum_{i=1}^n u_{ij} [q_i^*(x, z) - q_i(x, t_i, h)]^2 \\ & + \sum_{j=1}^m \nu_j \sum_{i=1}^n u_{ij} [p_i^*(\theta) - p_i(\theta, h)]^2 \\ & + \sum_{j=1}^m \beta_j [b_j - h_j]^2 \end{aligned} \quad (6)$$

where the first term on the right-hand side represents deviations between measured and predicted space-time variables (e.g., observed pressure heads or moisture contents at different locations and/or times, or the cumulative infiltration rate versus time). In this term,  $m_j$  is the number of different sets of measurements, and  $n_j$  is the number of measurements in a particular measurement set. The  $q_i^*(x, z)$  terms represent specific measurements at time  $t_i$  for the  $j$ th measurement set at location  $x(z, z)$ ,  $q_i(x, t_i, h)$  are the corresponding model predictions for the vector of optimized parameters  $\mathbf{b}$  (e.g.,  $\theta_r$ ,  $\theta_s$ ,  $\alpha$ ,  $n$ , and  $K_h$ ), and  $\nu_j$  and  $u_{ij}$  are weights associated with a particular measurement set or point, respectively. The weighting factor,  $\nu_j$ , is given by the inverse of the number of measurements multiplied by the variance of those observations, and  $u_{ij}$  is equal to 1 in this case. The second term of Eq. (6) represents differences between independently measured and predicted soil hydraulic properties (e.g., soil-moisture characteristic,  $\theta(h)$ , hydraulic conductivity,  $K(\theta)$  or  $K(h)$ , and/or diffusivity,  $D(\theta)$  or  $D(h)$  data), whereas the terms  $m_j$ ,  $n_j$ ,  $\nu_j$ ,  $u_{ij}$ ,  $\beta_j$ ,  $p_i^*(\theta)$ , and  $u_{ij}$  have meanings similar to those in the first term but now stand for the soil hydraulic properties. The last term of Eq. (6) rep-

resents a penalty function for deviations between prior knowledge of the soil hydraulic parameters,  $b_j$ , and their final estimates,  $h_j$ , where  $n_j$  is the number of previously known parameters and  $\nu_j$  are their preassigned weights. Estimates made with prior information (such as those used in the third term of Eq. (6)) are known as Bayesian estimates (Bard 1974). Note that the covariance (weighting) matrices that provide information about the measurement accuracy, as well as any possible correlation between measurement errors and/or parameters, are assumed to be diagonal in this study.

Minimization of the objective function  $\Phi$  is accomplished by using the Levenberg-Marquardt nonlinear minimization method (Marquardt 1963), which combines the Newton and steepest descent methods.

## METHODS

### The Laboratory Aquifer

Prototype tests were conducted in a laboratory aquifer measuring 4.7 m  $\times$  4.7 m  $\times$  2.6 m. The aquifer material is a sandy soil with occasional kaolin pockets and is underlain by 20 cm of gravel. The average bulk density of undisturbed soil samples was determined to be 1.69 g/cm<sup>3</sup>. A description of the aquifer and its soil properties was presented previously by Gribb et al. (1998). To simulate field conditions with various soil moisture profiles, the water table in the laboratory aquifer was raised and/or lowered by pumping water in/out of a French drain system. The hydraulic properties of the soil were determined directly using several standard methods. Pressure plate (ASTM D-2434) capillary rise (Lambe 1951) and computer-automated extraction/sorption (Ray and Morris 1994) tests were performed to evaluate the drying and wetting branches of the retention curve, respectively (Singleton 1997). The nonlinear optimization program, RETC, (van Genuchten et al. 1991) was used to fit  $\theta(h)$  data to Eq. (4). Constant head permeability tests (ASTM D-2334) were performed in the lab, and slug (Bouwer and Rice 1976) and Guelph permeameter (Rynolds 1993) tests were performed in situ to determine  $K_s$  (Scaturro 1993; Singleton 1997).

### Test Procedure

Two sets of cone permeameter tests were performed by Leonard (1997). In the first case (Set A), the prototype was placed in a hole and the soil carefully backfilled around it (12/20/96).

In the second case (Set B), the cone permeameter was pushed continuously to the testing depth by a drill rig (01/28/97). These two sets of tests and results obtained via inverse modeling demonstrate the effects of placement on the optimized hydraulic properties of the soil.

The Set A tests were conducted with the cone permeameter after it was buried in the soil. The center of the screened section was 65 cm below the soil surface. The water table was raised to the soil surface and then drawn down 26 and 48 cm below ground surface before testing. Under these saturated conditions, the prototype was used as a piezometer, and Hvorslev's (1951) simple analytical equation was used to obtain saturated hydraulic conductivity values. The detailed test procedure and data analysis were described by Leonard (1997) and Gribb et al. (1998). The water table was then lowered to approximately 190 cm below the soil surface, and the laboratory aquifer was not disturbed for 2 weeks, after which time several tests were run under unsaturated conditions. This depth and length of time ensured that the tests would not be influenced by the water table and that low initial pressure heads would be detected by the tensiometers. Sufficiently low initial pressure heads were important so that a wide range of pressure heads would be monitored during a test and the corresponding moisture content range during data collection would be large enough to minimize nonuniqueness problems. It is necessary at this point to note that the initial pressure heads measured with both tensiometers were always higher than  $-50$  cm for all of the tests we conducted in the laboratory aquifer because it was not possible to induce lower heads. It is evident from our experience with this soil that the equivalent moisture content at this pressure head level is close to  $\theta_r$  and the corresponding low hydraulic conductivities constrain moisture redistribution in the laboratory aquifer. Similar circumstances were also dis-

cussed by Inoue et al. (1998), in numerical simulations of soil water extraction experiments in sandy soil, pressure heads did not decrease below  $-35$  cm, again because of the limiting function of low hydraulic conductivities near  $\theta_r$ .

Three representative tests were selected to demonstrate the influence of different applied pressure heads and variable initial pressure head readings on the course of the experiment and, finally, on the inverse solution results. A summary of the applied boundary condition and the observed initial and final conditions is shown in Table 1. Flow data were collected electronically every second for  $0 < t < 840$  s (Leonard 1997; Gribb et al. 1998). A water pressure head of 50 cm was supplied to the center of the screen for  $0 < t < 400$  s for the test performed on 01/06/97 (Test A1), and a water pressure head of 30 cm was applied for  $0 > t > 440$  s for the tests conducted on 01/25/97 (Test A2) and 01/26/97 (Test A3). After the source of water was terminated, data collection continued as the water redistributed in the flow regime. Only inflow and pressure head data obtained at 5-s intervals for  $0 < t < 400$  s were used to obtain wetting branches of the soil hydraulic characteristics with the parameter estimation procedure.

The second set of tests (Set B) was conducted with the prototype after it was pushed into the soil to a depth of 70 cm. In this case, tests were run under unsaturated conditions immediately after placement of the prototype. Two representative tests run with the same applied water pressure heads as used for Set A are shown here to demonstrate again the influence of different applied boundary conditions (Table 1). However, in the case of Set B, we chose tests performed in the reverse order to illustrate that the soil hydraulic properties changes attributed to the applied pressure head differences in Set A (see Discussion) were not attributable to repetition of experiments in the same location. As with the previous

TABLE 1  
Applied boundary conditions and observed initial and final conditions for Set A and B tests

Test	Date	Initial pressure head readings (cm)		Final pressure head readings (cm)		Final cumulative infiltration volume (cm <sup>3</sup> )
		Lower tensiometer	Upper tensiometer	Lower tensiometer	Upper tensiometer	
A1	01/06/97	52.5	-47.6	-50.2	-15.9	11217
A2	01/25/97	32.5	-47.4	-49.3	-18.3	6368
A3	01/26/97	32.5	-30.2	-31.5	-20.0	5438
B1	01/30/97	32.5	-45.1	-48.4	-22.3	2222
B2	01/31/97	52.5	-45.0	-46.7	-10.8	5438

set of tests, flow are electronically collected every second for  $v < l < 840$  s (Leonard 1997). A water pressure head of 30 cm was supplied to the center of the screen for  $0 < t < 540$  s for the test performed on 01/30/97 (Test B1), and a water pressure head 50 cm was supplied for  $0 < t < 440$  s for the test conducted on 01/31/97 (Test B2). Pressure head and inflow data obtained at 5-s increments for  $0 < t < 400$  s were used in the parameter estimation problem.

#### Inverse Solutions

Initial conditions for modeling were based on the pressure head conditions in the laboratory aquifer. For Set A, the water table was 189 cm, 183 cm, and 184 cm below ground surface for Tests A1, A2, and A3, respectively. However, the initial readings of the lower and upper tensionmeters were as follows: Test A1: -47.6 cm and -50.2 cm; Test A2: -47.4 cm and -49.3 cm; and Test A3: -30.2 cm and -31.5 cm (Table 1). The volumetric moisture contents of soil samples taken at the same depth as the tensionmeters were determined to be approximately 8% for all three tests. Because the soil moisture distribution was neither hydrostatic nor uniform before testing, the pressure heads in the domain were set equal to the upper tensionmeter initial reading for elevations at or above that of the upper tensionmeter. Similarly, the pressure heads at the elevation of the lower tensionmeter or below were set equal to the lower tensionmeter initial reading. The pressure heads between the two tensionmeters were distributed linearly. The external boundaries were set as no-flow boundaries. The screen was modeled as a constant head boundary, with water pressure heads ranging from 52.5 to 47.5 cm (or 32.5 to 27.5 cm) from bottom to top for  $0 < t < 400$  s. The water table was 183 cm and 184 cm below ground surface for Tests B1 and B2, respectively. In the case of Set B, initial readings of lower and upper tensionmeters were as follows: Test B1: -45.1 cm and -48.4 cm; Test B2: -45.0 cm and -46.7 cm (Table 1). The soil moisture distribution was again neither hydrosta-

tic nor uniform before testing, and, thus, the initial and boundary conditions were set up similarly to the previous examples.

Four inverse solutions for each test of Set A were obtained with different sets of parameters to be optimized and different inputs to the objective function. A summary of the solutions is shown in Table 2. In the first case, Inverse Solution 1 yielded estimates of the unknown parameters,  $\alpha$ ,  $n$ ,  $\theta$  and  $K$ , for  $\theta = 0.008$  cm<sup>3</sup>/cm<sup>3</sup>. In the second case, Inverse Solution 2 yielded estimates of  $\alpha$ ,  $n$ , and  $K$ , for  $\theta = 0.35$  cm<sup>3</sup>/cm<sup>3</sup> and  $\theta = 0.008$  cm<sup>3</sup>/cm<sup>3</sup>. In the third and fourth cases, additional moisture content information was introduced into the objective function to find a more realistic estimate of  $\theta$ . For Inverse Solution 3, the impact of a  $\theta(h)$  point given by the initial soil moisture content paired with the initial reading of the upper tensionmeter was investigated using the second term of Eq. (6). Solely for Inverse Solution 4, the moisture contents of undisturbed soil samples taken at locations corresponding to nodes shown in Fig. 3, were determined at 400 s, 380 s, and 340 s for tests A1, A2, and A3, respectively, and this information was included in the first term of Eq. (6). Inverse Solutions 3 and 4 yielded estimates of the unknown parameters,  $\alpha$ ,  $n$ ,  $\theta$  and  $K$ , for  $\theta = 0.008$  cm<sup>3</sup>/cm<sup>3</sup>. As a result of nonconvergence, we were unsuccessful in our attempts to obtain estimates of the unknown parameters when initial and final moisture content data were included in the objective function.

For Set B tests, soil moisture contents were not measured because of difficulties encountered while trying to sample the well consolidated soil. Therefore, we executed Inverse Solutions 1 and 2 on these data sets. In addition, because the initial tensionmeter readings for tests B1 and B2 were similar to those for Set A, an initial soil moisture content of 8% was paired with the pressure head reading of upper tensionmeter, and Inverse Solution 3 was also performed.

In all cases, the value of the residual moisture content was set equal to the value obtained from capillary rise experiments inasmuch as we

anticipated that it would not be identifiable from the near-saturated tests. The first estimate of the optimized parameters had to be set within a realistic range of values characteristic for sandy soil to obtain reasonable final estimates. Use of extreme values for the initial estimates resulted in nonconvergence of the solution. The finite element mesh shown in Fig. 3 has been designed specifically for sandy soil. It should be noted that the mesh must be reconstructed for different soil types to ensure numerical stability of the solution.

## RESULTS AND DISCUSSION

### Comparison of Inverse Solutions

Results of the optimization processes and the representative hydraulic property values obtained using standard techniques are shown in Figs. 4 through 12 and Tables 3, 4 through 5. Measured and simulated cumulative flow and pressure head data in time from the five inverse solutions are plotted in Figs. 4-8. The estimated retention curves for all solutions of the five cone permeameter tests, along with those determined independently with capillary rise, the extraction/sorption, and pressure plate tests (Singleton 1997) are presented in Figs. 9 through 12. Table 3 shows the hydraulic parameters  $\alpha$ ,  $n$ ,  $\theta$  and  $K$  estimated via inverse modeling. Table 4 contains the hydraulic parameters  $\alpha$ ,  $n$ ,  $\theta$ , and  $\theta$  of the retention curves

TABLE 2  
Summary of inverse solutions IS1-IS4 for Set A and Set B tests.

Inverse solution	Optimized parameters	Additional inputs
IS1	$\alpha, n, \theta, K$ , for $\theta = 0.008$	
IS2	$\alpha, n, K$ , for $\theta = 0.35$ and $\theta = 0.008$	Point of the retention curve $\theta(h)$
IS3	$\alpha, n, \theta, K$ , for $\theta = 0.008$	
IS4	$\alpha, n, \theta, K$ , for $\theta = 0.008$	Soil moisture contents at different times and locations

TABLE 3  
Hydraulic parameter estimates obtained from inverse solutions IS1-IS4 for Set A and Set B tests

Test	Hydraulic Parameters ( $\theta = 0.008$ cm <sup>3</sup> /cm <sup>3</sup> )			
	$\alpha$ (cm <sup>-1</sup> )	$n$ (-)	$\theta$ (cm <sup>3</sup> /cm <sup>3</sup> )	$K$ (cm-sec <sup>-1</sup> )
A1, IS1	0.05244	6.999	0.463	0.01188
A1, IS2	0.04496	5.762	0.350	0.01135
A1, IS3	0.0445	3.484	0.500	0.01219
A1, IS4	0.04083	5.994	0.282	0.01102
A2, IS1	0.05729	7.000	0.432	0.00960
A2, IS2	0.05020	5.998	0.350	0.00908
A2, IS3	0.0509	3.105	0.482	0.01024
A2, IS4	0.04795	3.015	0.374	0.00920
A3, IS1	0.04351	6.281	0.249	0.00806
A3, IS2	0.03993	4.290	0.350	0.00821
A3, IS3	0.0429	5.826	0.261	0.00808
A3, IS4	0.04470	5.269	0.315	0.00821
B1, IS1	0.0474	4.242	0.205	0.00321
B1, IS2	0.03896	2.124	0.350	0.00356
B1, IS3	0.0434	2.742	0.261	0.00339
B2, IS1	0.04755	7.000	0.221	0.00394
B2, IS2	0.03561	2.568	0.350	0.00418
B2, IS3	0.03895	3.186	0.277	0.00408

TABLE 4

Soil-moisture retention curve parameters obtained with standard laboratory methods

Test Method	Hydraulic parameters		
	$\alpha$ (cm <sup>-1</sup> )	$n$ (-)	$\theta$ (cm <sup>3</sup> /cm <sup>3</sup> )
Extraction/Sorption (wetting curve) (Singleton 1997)	0.139	2.17	0.042
Capillary rise (wetting curve) (Singleton 1997)	0.086	3.60	0.008
Pressure plate (drying curve) (Singleton 1997)	0.045	1.61	0.008
			0.35

TABLE 5

Results of laboratory and in situ tests for saturated hydraulic conductivity

Test method	Mean $K_s$ value, (cm-sec <sup>-1</sup> )
Constant head inflow test, cone permeameter (Leonard 1997)	0.0134
Slug tests (Scaturro 1993)	0.00725
Guelph permeameter tests (Singleton 1997)	0.0199
Guelph permeameter tests (Scaturro 1993)	0.00349
Constant head test, laboratory samples (Singleton 1997)	0.00385

obtained with standard laboratory methods. Finally, Table 5 presents the mean values of  $K_s$  determined with the cone permeameter under saturated conditions (Leonard 1997; Gribb et al. 1998), slug tests (Scaturro 1993), Guelph permeameter tests (Scaturro 1993; Singleton 1997), and laboratory constant head tests (Singleton 1997).

Estimates of the parameters  $\alpha$ ,  $n$ ,  $\theta$ , and  $K$  obtained solely from the cone permeameter flow responses (Inverse Solution 1) provided the best fit of measured data for all numerical solutions, as expected, because there were no additional constraints (such as moisture contents) on the inverse solution. The resulting retention curves for all tests had the same shape except for pressure heads near zero, where the shapes of the characteristic curves were strongly influenced by the value of  $\theta$ . The resulting saturated moisture content,  $\theta_s$ , was also either much higher (Tests A1 or A2), or much smaller (Tests A3, B1, and B2) than that obtained from the laboratory test methods.

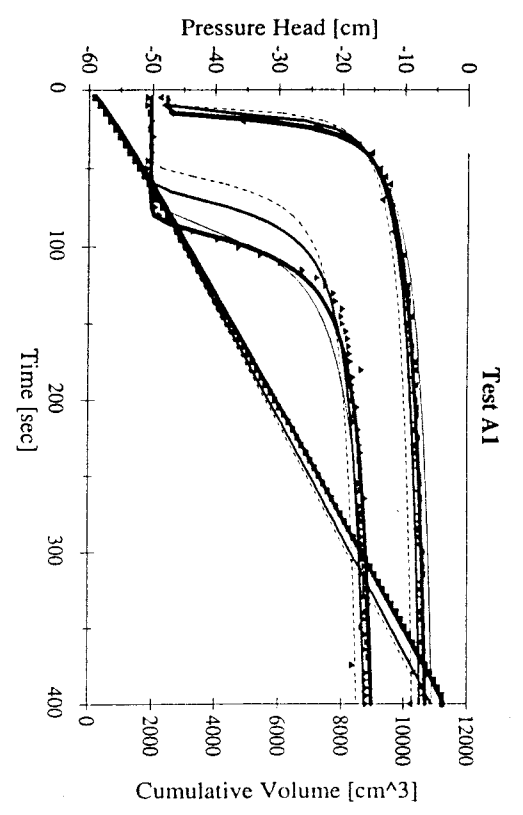


Fig. 4. Measured and simulated flow responses for Test A1.

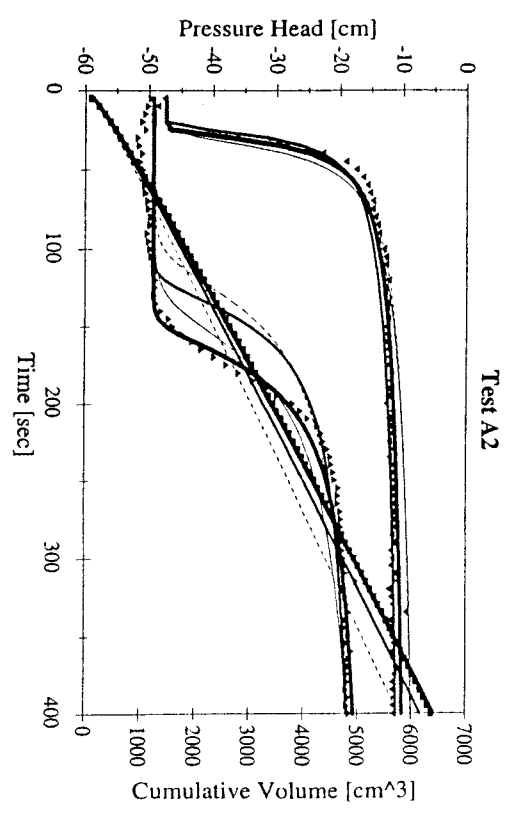


Fig. 5. Measured and simulated flow responses for Test A2.

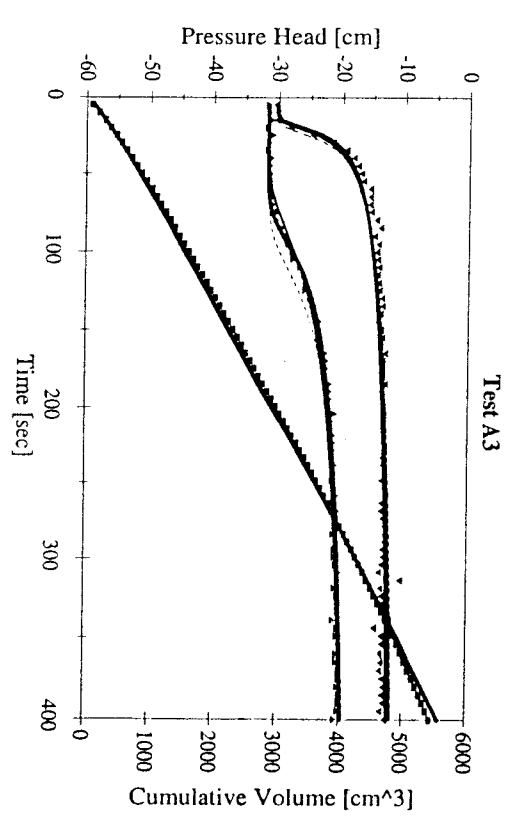


Fig. 6. Measured and simulated flow responses for Test A3.

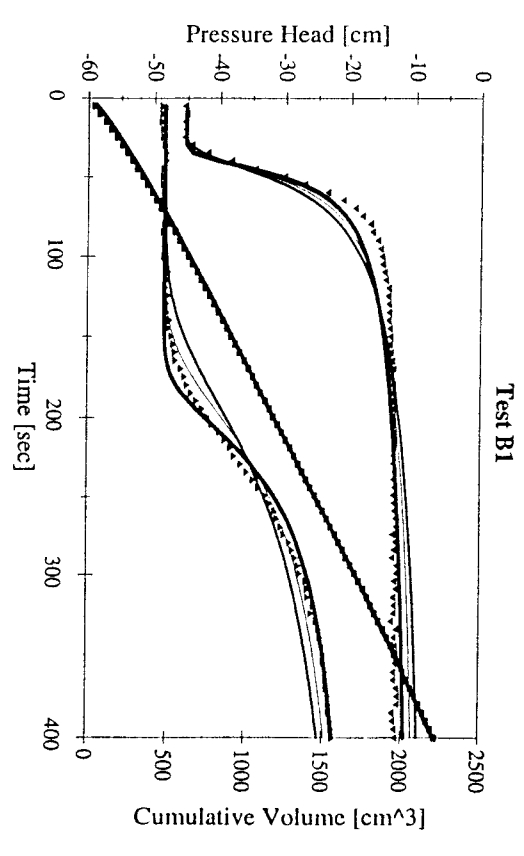


Fig. 7. Measured and simulated flow responses for Test B1.

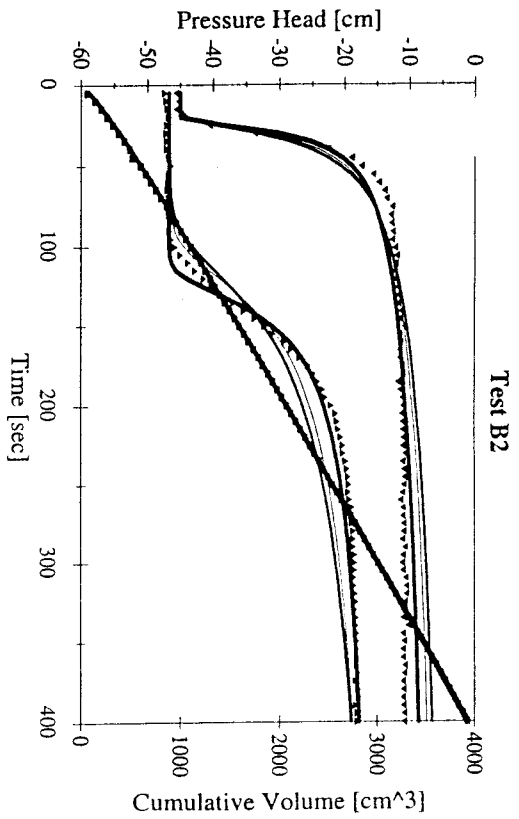


Fig. 8. Measured and simulated flow responses for Test B2.

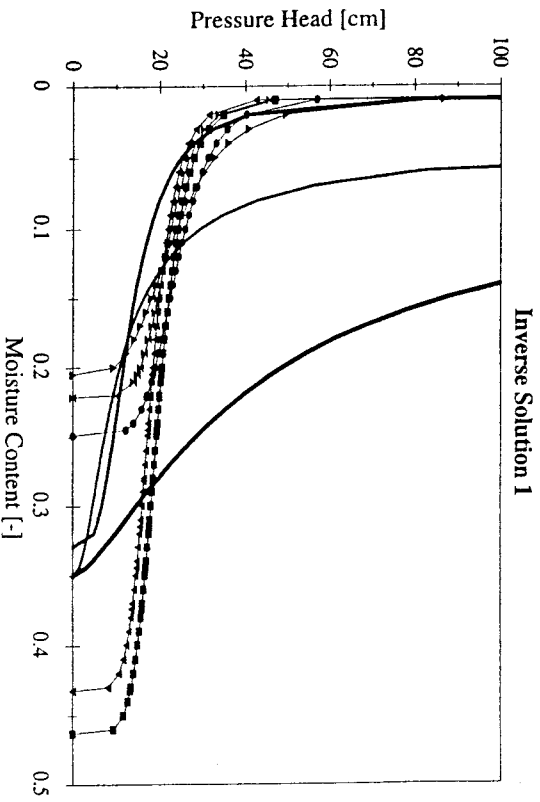


Fig. 9. Soil-moisture retention curves obtained with standard methods and those obtained from Inverse Solution 1 for Tests A1, A2, A3, B1, and B2.

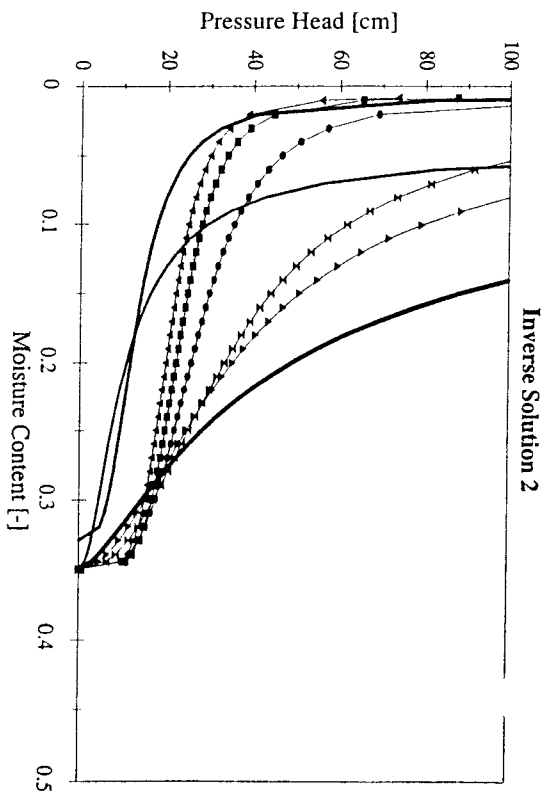


Fig. 10. Soil-moisture retention curves obtained with standard methods and those obtained from Inverse Solution 2 for Tests A1, A2, A3, B1, and B2.

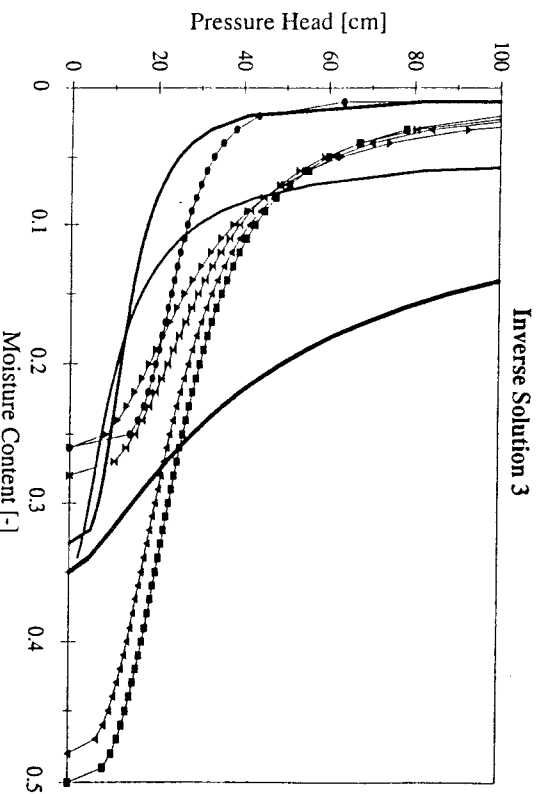


Fig. 11. Soil-moisture retention curves obtained with standard methods and those obtained from Inverse Solution 3 for Tests A1, A2, A3, B1, and B2.

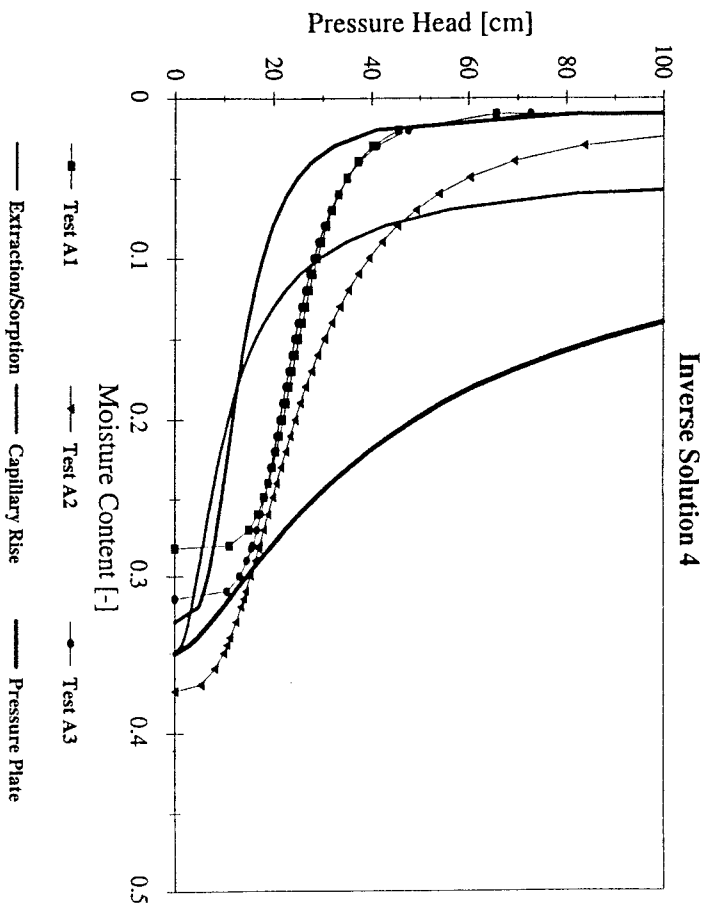


Fig. 12. Soil-moisture retention curves obtained with standard methods and those obtained from Inverse Solution 4 for Tests A1, A2, and A3.

Nonuniqueness of  $\theta$  values was anticipated by the results of earlier numerical experiments with error-free synthetic data. Grubb (1996) showed that an objective function similar to Eq. (6) was sensitive to  $K_r$  and  $\alpha$  but not sensitive to  $\theta$ , and  $n$ . Because  $\theta$  was not estimated reliably for this test, Inverse Solution 2 was performed to investigate the influence of  $\theta$  on the other parameter estimates. In this case,  $\theta$  was set equal to  $0.35 \text{ cm}^3/\text{cm}^3$  and parameters  $\alpha$ ,  $n$ , and  $K_r$  were optimized. The simulated cumulative flow data either underestimated reality slightly when  $\theta$  resulting from Inverse Solution 1 was higher than  $0.35 \text{ cm}^3/\text{cm}^3$  (Tests A1 and A2) or fit the measured data very well when  $\theta < 0.35 \text{ cm}^3/\text{cm}^3$  (Tests A3, B1 and B2). The underestimation of the cumulative flow volume was a consequence of the simulated low capacity of the soil for inhibition of water. It seems that the real flow regime was influenced by anisotropy and heterogeneities caused by backfilling or the applied Eggs (4) and (5), with  $l = 0.5$ , are not accurate enough

to describe the shapes of both hydraulic properties. As it is beyond the scope of this study, we will concentrate on this problem in our next series of experiments. The modeled pressure heads at the position of the lower tensiometer tracked closely to observed data for all tests. However, the simulated pressure heads at the position of the upper tensiometer showed earlier progressions of the wetting front when compared with the measured data for all tests. Similar to the influence of  $\theta$  on cumulative flow data discussed previously, the course of pressure head changes in time was steeper (if  $\theta$ , from Inverse Solution 1 was higher than  $0.35 \text{ cm}^3/\text{cm}^3$ ) or more gradual (if  $\theta < 0.35 \text{ cm}^3/\text{cm}^3$ ) than the observed data. However, the final simulated and observed pressure head responses were almost the same in all cases. Comparison of the soil hydraulic properties resulting from Inverse Solutions 1 and 2 shows that the optimized parameter  $K_r$  seems to be correlated to  $\theta$ , as well, e.g.,  $K_r$  is larger/smaller with larger/smaller  $\theta$ , whereas  $\alpha$  and  $n$  are always smaller for

Inverse Solution 2 (e.g., the retention curves have more gradual slopes). In addition, the lower the value of  $\theta$  obtained from Inverse Solution 1, the more gradual the retention curve returned from Inverse Solution 2. It must be noted that  $\theta$  can be optimized instead of  $\theta$  with  $\theta$  set equal to a laboratory derived value ( $0.35 \text{ cm}^3/\text{cm}^3$ ). In cases in which Inverse Solution 1 returned  $\theta$  values less than  $0.35$ ,  $\theta$  will be equal to  $0.35 - \theta_{\text{ISI}} + \theta_{\text{ISI}}$ , (where  $\theta_{\text{ISI}}$ ,  $\theta_{\text{ISI}}$  are from Inverse Solution 1). The values of  $\alpha$ ,  $n$  and  $K_r$ , as well as the simulated flow responses, will remain unchanged. The only effect will be a shift of the Inverse Solution 1  $\theta(h)$  curve along the  $\theta$  axis. However, this does not improve soil hydraulic property estimates in our examples because the resulting residual moisture content would be too high. For cases with  $\theta$  higher than  $0.35 \text{ cm}^3/\text{cm}^3$ , the resulting  $\theta$  would have the tendency to go to zero and the inversion results would be similar to those of Inverse Solution 2.

For Inverse Solutions 3 and 4, the four parameters  $\alpha$ ,  $n$ ,  $\theta$  and  $K_r$  were estimated. Additional soil moisture data were used in the optimization process to obtain more realistic values of  $\theta$ . Initial moisture content data were used for Inverse Solution 3, whereas final moisture content data were added to the objective function for Inverse Solution 4.

The fit of measured data resulting from Inverse Solution 3 is better than for Inverse Solution 2. The cumulative flow volumes were predicted very well, but the simulated tensiometer responses increased more gradually than the measured values for all tests. The fixed point of the retention curve for all tests. The fixed point of the retention curve close to residual values resulted in higher values of  $\theta$  and  $K_r$  and lower values of  $\alpha$  and  $n$ . Correlation between  $\theta$  and  $K_r$  values, as discussed previously, was again evident. As mentioned before, approximately the same initial moisture contents were observed for different initial tensiometers readings. This is especially curious in the case of Test A3. We expected higher initial moisture contents because the surface of the soil was lightly irrigated overnight before testing, and the initial tensiometer readings were correspondingly higher. It seems for a wetting process of this type, the retention curve follows a shape similar to that determined with the computer-automated extraction/sorption method. The resulting courses of the retention curves show clearly the effect of fixed points of the retention curve, i.e., the influence of the initial conditions that produce different scanning retention curves.

Inverse Solution 4 yielded the worst fit of

measured data of all four types of solutions because of constraints imposed by the final moisture content inputs. However, the soil moisture inputs for higher levels of saturation resulted in estimates of  $\theta$  similar to values obtained with standard laboratory techniques. Compared with Inverse Solution 1, the hydraulic parameters changed in a manner similar to that of Inverse Solutions 2 and 3. However, there was no correlation among the results from the four different numerical analyses because of different priorities in the objective function, and, consequently, the optimization process. In addition, the shapes of the retention curves were influenced strongly by the moisture content values  $\theta$  which were paired with simulated pressure heads,  $h$ . These  $\theta$  values were determined at different times and at variable positions with respect to the water source for different levels of soil saturation.

Review of the fits of the four inverse solutions for all five tests shows that there is very good coincidence of measured and simulated data for all solutions for Test A3. However, different soil hydraulic properties were obtained from these solutions. Such nonuniqueness is probably caused by the smaller range of pressure head changes detected during this test compared with the other tests.

Comparison of the estimated parameters from the four inverse solutions for each test shows that the values of  $K_r$  obtained are almost the same for all numerical analyses. Values of  $\alpha$  are similar for all inverse solutions as well, whereas values of  $\theta$  and  $n$  vary.

#### *Influence of Applied Pressure Head at the Source and Initial Conditions*

The influence of applied pressure heads on the course of the experiment and, finally, on the optimization results, is possible to see via comparison of tests with similar initial tensiometer readings: Set A, Test A1 (applied pressure head  $h_0 = 52.5 \text{ cm}$ ) versus Test A2 ( $h_0 = 32.5 \text{ cm}$ ), and Set B, Test B1 ( $h_0 = 32.5 \text{ cm}$ ) versus Test B2 ( $h_0 = 52.5 \text{ cm}$ ). In both cases, numerical analysis of data from tests with lower applied pressure heads resulted in lower optimized values of  $\theta$  and  $K_r$  than those estimated from tests run with higher applied pressure heads. For instance, in the case of Inverse Solution 1, the optimized saturated moisture contents,  $\theta$ , were  $0.463 \text{ cm}^3/\text{cm}^3$  (Test A1) versus  $0.432 \text{ cm}^3/\text{cm}^3$  (Test A2) and  $0.205 \text{ cm}^3/\text{cm}^3$  (Test B1) versus  $0.221 \text{ cm}^3/\text{cm}^3$  (Test B2). The saturated hydraulic conductivities,  $K_r$ , were  $0.01188 \text{ cm/s}$  (Test A1) versus  $0.00960$

cm/s (Test 1) 0.00321 cm/s (Test B1) versus 0.00394 cm/s (Test B2). The effect of applied pressure head on  $\theta$  and  $K_s$  is evident for all other solutions and other tests (see Table 3) with the exception of Inverse Solution 2 ( $\theta$  value fixed) and Inverse Solution 4. The results of Inverse Solution 4 were influenced predominantly by the additional soil moisture content data and, as such, the effect of applied pressure head could not be detected. We simulated the courses of Test A2 and B1 with the hydraulic parameters obtained from Inverse Solution 1 for Test A1 and B2, respectively. Only the final pressure heads and total cumulative infiltration volumes are shown in Table 6. The simulated final pressure heads were similar to final readings of the tensiometers (Table 1). On the other hand, the predicted final cumulative infiltration volumes were noticeably lower than the observed volumes (Table 1). We suspect that the lower infiltration capacity indicated from tests with lower applied pressure heads was caused by encapsulated air in the soil or the occurrence of preferential flow, which can be different for different pressure conditions. Investigation of these effects would require continuous measurement of soil moisture and/or pressure heads at different locations in the domain during testing.

The influence of initial conditions can be examined on examples with the same applied pressure heads ( $h_0 = 32.5$  cm), but with different initial readings of the lower and upper tensiometers, as with Tests A2 and A3 ( $-47.4$  cm and  $-49.3$  cm versus  $-30.2$  cm and  $-31.5$  cm). In the case of higher initial pressure head readings, optimized values of  $\theta$  and  $K_s$  were lower than those obtained for tests with lower initial pressure head readings. For Inverse Solution 1, the optimized saturated moisture contents,  $\theta_s$ , were 0.432 cm<sup>3</sup>/cm<sup>3</sup> (Test A2) versus 0.249 cm<sup>3</sup>/cm<sup>3</sup> (Test A3) and saturated hydraulic conductivities,  $K_s$ , were 0.000960 cm/s (Test A2) versus 0.000806 cm/s (Test A3). The effect of initial pressure heads on  $\theta$  and  $K_s$  is evident for all solutions (Table 3) with the exception of Inverse Solutions 2 and 4. We simulated the course of Test A3 with hydraulic parameters from Inverse Solution 1 for Test A2 (Table 6). We obtained a higher final cumulative infiltration volume than the observed volume, which indicated lower infiltration capacity as described previously. The flow regime also seemed to be affected by hysteresis of the soil hydraulic properties as the final pressure heads were also overestimated.

An analysis of results for Set A shows a decreasing trend of optimized  $K_s$  values with decreasing initial induced hydraulic gradients. In addition, these  $K_s$  values are lower than the mean value of  $K_s$  (0.0134 cm/s) found from cone penetrometer tests performed under saturated conditions (Gribb et al. 1998). As a result, a higher applied pressure head is more likely to yield estimates of  $K_s$  closer to those obtained under saturated conditions. On the other hand, a lower applied pressure head is desirable because the rates at which pressure heads increase at the tensiometers are slower, providing more information for more accurate determination of  $\alpha$  and  $n$ . However, from a practical point of view, the optimized  $K_s$  values are not so different from each other, and we believe that the investigated range of 32.5 to 52.5 cm of applied pressure head is suitable for this type of soil. Finally, as already required for uniqueness of the inverse solutions, initial pressure heads in the soil should be as low as possible.

#### Comparison of Results for Sets A and B and Comparison of the Soil Hydraulic Characteristics Obtained via Inverse Solution and Other Techniques

The results of the two sets of tests presented here are noticeably different. The main influences are likely to be (i) disturbance of soil surrounding the cone (such as densification) caused by pushing or (ii) structural differences between the backfilled and well consolidated soil. It seems that soil structure changes caused by direct push are not dominant for this soil at the investigated

depth. The results of both sets are within the range of hydraulic properties obtained with other standard tests performed in situ and in the laboratory on different types of samples. The characters of the retention curves obtained from Inverse Solution 1 for all tests are most similar to the retention curve determined from capillary rise tests conducted on repacked soil columns. For Tests A1 and A2, the first parts of the retention curves are essentially the same for all other solutions despite largely varying  $\theta_s$  values. The retention curves found for Tests A3, B1, and B2 with  $\theta_s$  fixed or estimated close to 0.35 cm<sup>3</sup>/cm<sup>3</sup> have more gradual shapes and lie between the two limiting branches of the retention curves obtained from capillary rise tests (wetting) and pressure plate tests (drying) performed on undisturbed samples. The shapes of the retention curves close to the limiting or scanning wetting branches of the retention curves were expected because of the wetting character of the experiment and also because  $\theta$  was set equal to 0.008 cm<sup>3</sup>/cm<sup>3</sup>. In general, the optimized saturated water contents for the pushed cone tests tended to be lower than for the buried cone test.

Saturated hydraulic conductivities obtained for the buried cone permeameter (Set A) are similar to  $K_s$  values obtained with the Gueph permeameter (Singleton 1997) and with slug tests (Scaturro 1993), whereas  $K_s$  values for the pushed cone permeameter (Set B) are closer to those obtained by Scaturro (1993) with the Gueph permeameter and to laboratory constant head tests (Singleton 1997). On average, the saturated hydraulic conductivity for the buried cone tests were two to three times higher than those for the pushed cone. For most practical applications, such differences would be acceptable. Although it is possible that these decreases (as with  $\theta$ ) were attributable to lower porosity close to the cone as a result of densification during pushing to the test depth, it must be noted that results tests performed in a consolidated soil profile are influenced highly by soil heterogeneity. Thus natural consolidation, the presence of clay particles, and pseudocoaggregation may have influenced the data obtained from the pushed cone permeameter more than that obtained with the buried one.

## CONCLUSIONS

We have presented here a modified cone penetrometer tool for simultaneous determination of the soil-moisture characteristic and hydraulic conductivity curves in unsaturated soil. We studied the influence of applied boundary and initial conditions on the final stage and on the estimated values of parameters describing soil hydraulic properties. We found that higher applied pressure head and lower initial pressure head conditions resulted in higher final pressure head readings and, consequently, higher values of  $\theta$  and  $K_s$ . We examined the impact of the different sets of optimized parameters and various inputs to the objective function. Logically, the numerical analysis of the cone permeameter flow responses alone provided the best fits of measured data, but values of  $\theta$  were not estimated reliably. On the other hand, restriction of  $\theta$  or improved estimates of  $\theta$  using additional soil moisture content information yielded worse fits of observed data. We see several ways to improve the solution. First, as discussed for the tests when the cumulative flow volumes were underestimated, the effect of anisotropy can be introduced as an additional parameter to be optimized. Secondly, the pore-connectivity parameter,  $l$ , in Eq. (5) can also be optimized to increase flexibility of the hydraulic conductivity curves, which in this case will probably yield sharper decreases in  $K(\theta)$  with decreasing moisture content. For other tests, when  $\theta$  values were too low, the importance of either fixing values of  $\theta$  or  $\theta_s$  at reasonable values or adding soil moisture content information to find the appropriate position of the retention curve is evident. However, our results in a laboratory aquifer composed of sandy soil showed that the saturated hydraulic conductivity was well estimated. For cases in which the saturated moisture content was estimated or fixed near the laboratory-derived value, the soil-moisture characteristic curves were between the wetting and drying curves obtained from other standard laboratory methods. Differences in the optimized parameter  $\alpha$  and, to some degree,  $n$ , within both experimental sets were minimal.

In this study, we analyzed only the wetting part of the cone experiment so that the wetting branches of soil hydraulic properties were obtained. We found that our method predicts values of  $K_s$  and  $\alpha$  very well and that the others are reasonably estimated. The soil extraction method of Inoue et al. (1998) is a comparable technique, with a similar flow domain but the reverse flow process used to obtain the drying branches of the soil hydraulic properties. This method was found to be more sensitive to  $n$  and  $\theta$  but not suitable for prediction of  $K_s$  and  $\alpha$  for sandy soils. There is no simple technique other than this method for numerical estimation of drying curves in situ in real time.

TABLE 6  
Simulated final conditions for Tests A2, A3 and B1

Simulated experiment	Source of hydraulic parameter inputs	Final pressure heads (cm)		Final cumulative infiltration volume (cm <sup>3</sup> )
		Lower tensiometer	Upper tensiometer	
Test A2	Test A1, IS1	-10.8	-18.3	8071
Test A3	Test A2, IS1	-10.0	-17.8	6395
Test B1	Test B2, IS1	-13.2	-22.7	2469



From inspection of the wetting and redistribution phases of cone permeameter tests (see Gribb et al. (1998)), it seems that it should be possible to obtain estimates of the both the wetting and drying curves. This will be our next subject of study.

It is well known that the soil fabric is disturbed as a result of cone placement; however, the results of these few tests in sandy soil show little impact of disturbance on the values of  $K_s$  or  $\alpha_n$ , and  $\theta$  returned from the optimization procedure. It is likely that disturbance effects will be more significant in other soil types. This will be also studied further in the field.

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